CEE598 - Visual Sensing for Civil Infrastructure Eng. & Mgmt.

Session 8- Linear Filters
From Spatial Domain to Frequency Domain

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Outline

What we have learned so far:

- Review of lighting
- Reflection and absorption
- What is image filtering and how do we do it?
- Color models (if time allows)

- Today:
  - Fourier transform and frequency domain
    - Frequency view of filtering
    - Sampling

Reading: [FP] Chapters 7,8

Some slides in this lecture are courtesy to Prof. S. Savarese, Prof. D. Hoiem, prof F. Li, prof S. Lazebnik, and various other lecturers

Review: image filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

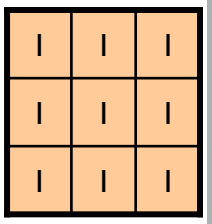
$h[.,.]$

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

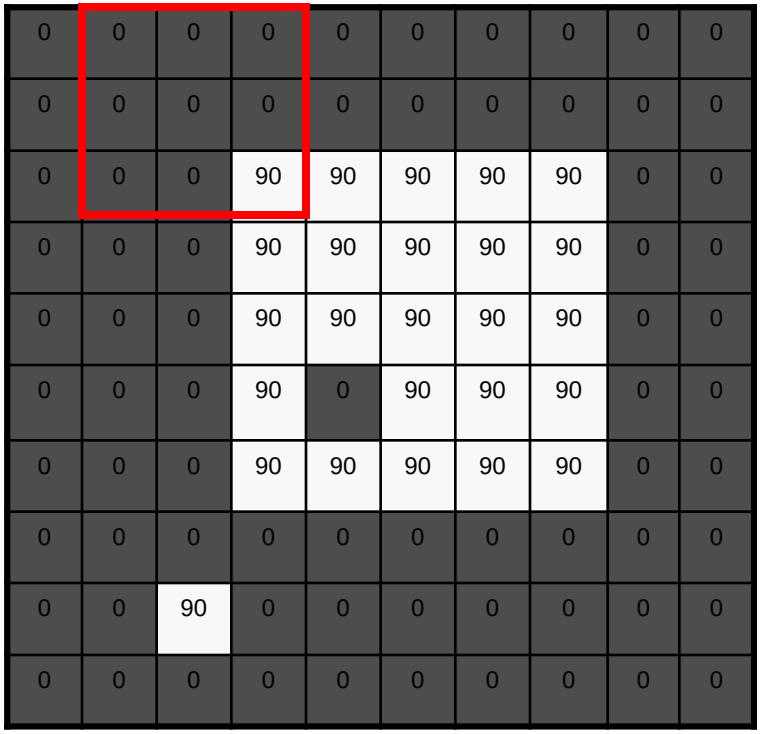
Credit: S. Seitz

Review: image filtering

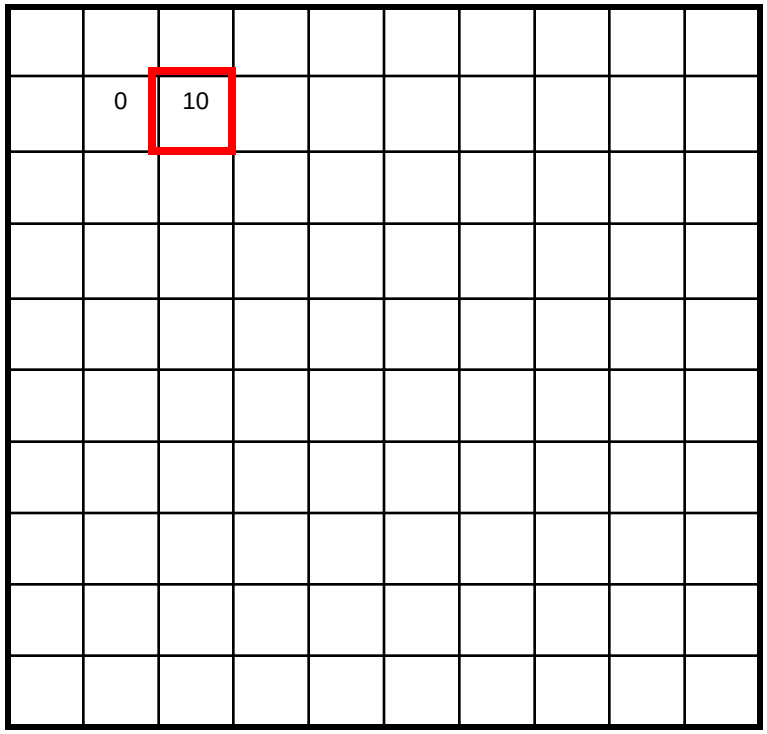
$g[\cdot,\cdot]$ $\frac{1}{9}$



$f[.,.]$



$h[.,.]$

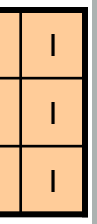


$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k,n+l]$$

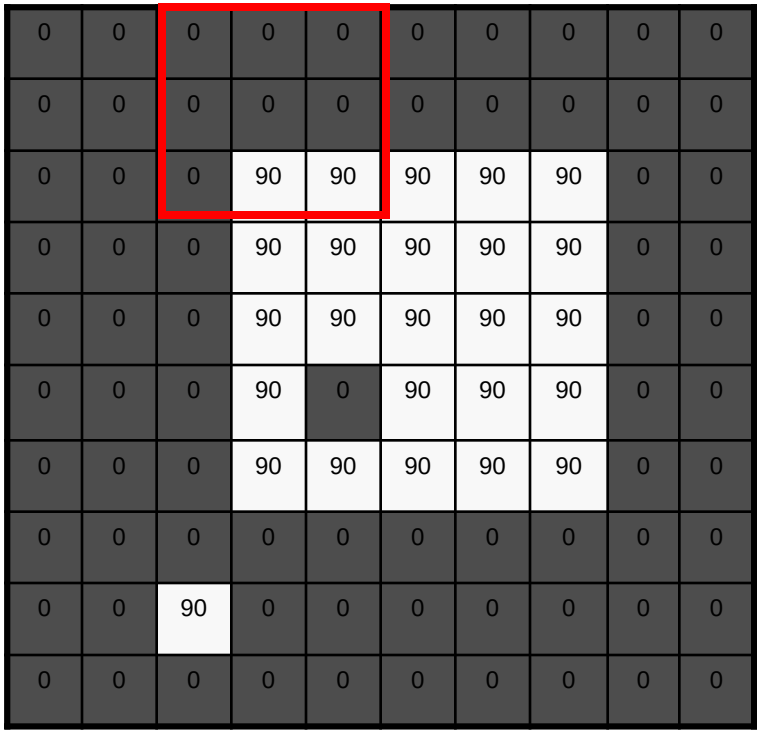
Credit: S. Seitz

Review: image filtering

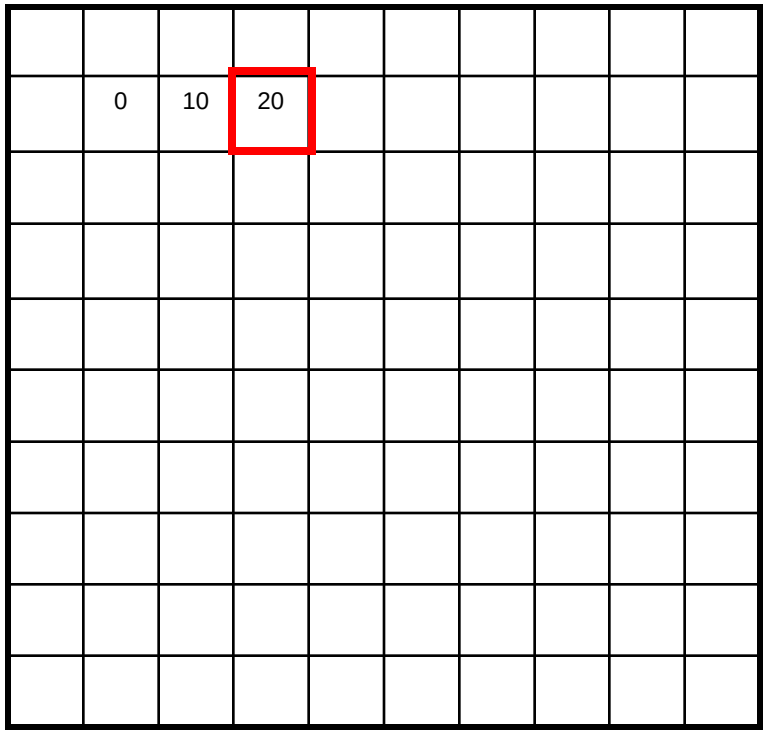
$g[\cdot,\cdot]$ $\frac{1}{9}$



$f[.,.]$



$h[.,.]$

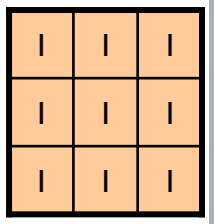


$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Review: image filtering

$$g[\cdot,\cdot]\ \frac{1}{9}$$



$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

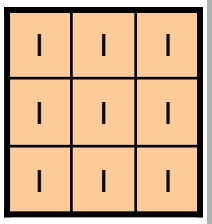
	0	10	20	30					

$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Review: image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$



$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

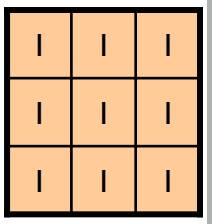
	0	10	20	30	30				

$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Review: image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$



$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

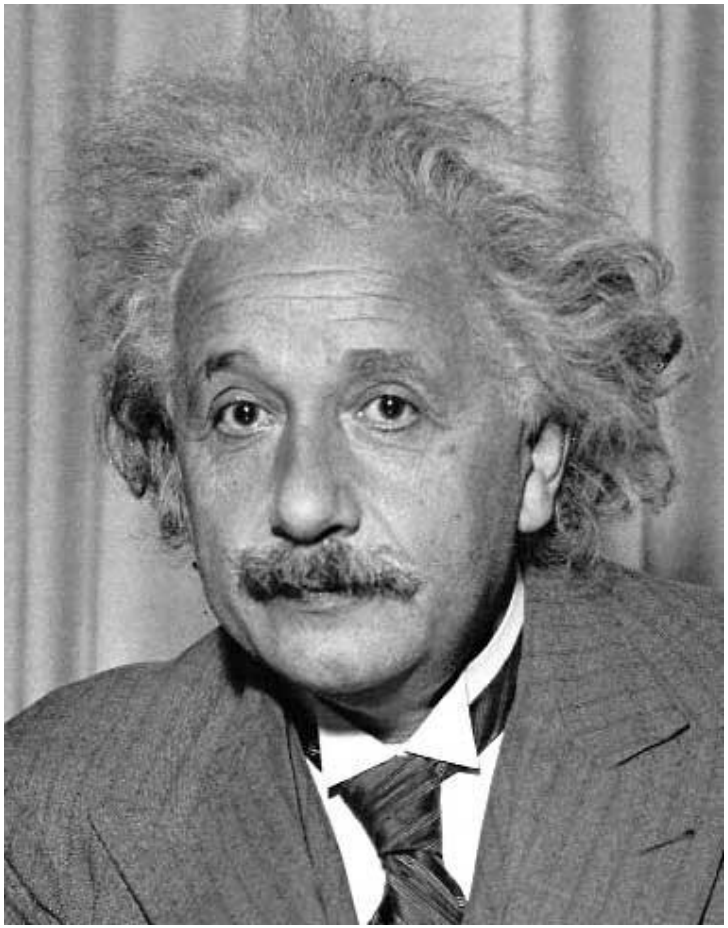
$h[.,.]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k,n+l]$$

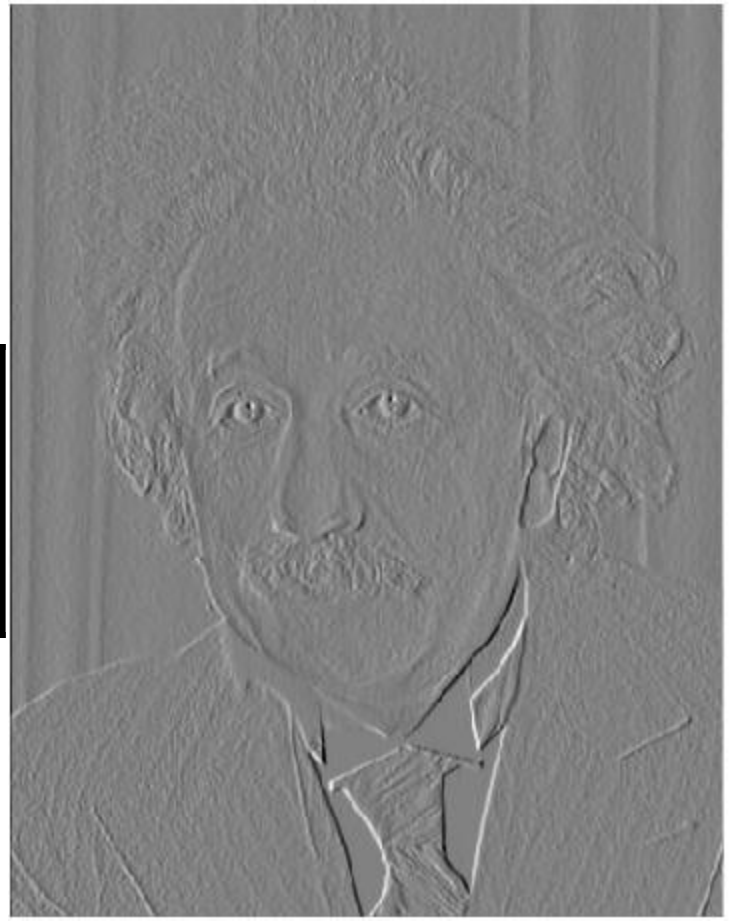
Credit: S. Seitz

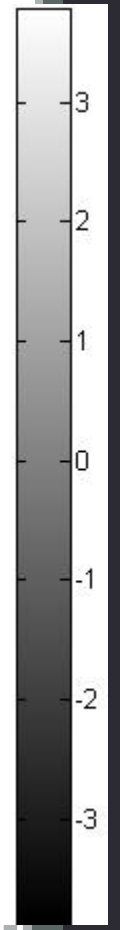
Review: image filtering in spatial domain



1	0	-1
2	0	-2
1	0	-1

Sobel

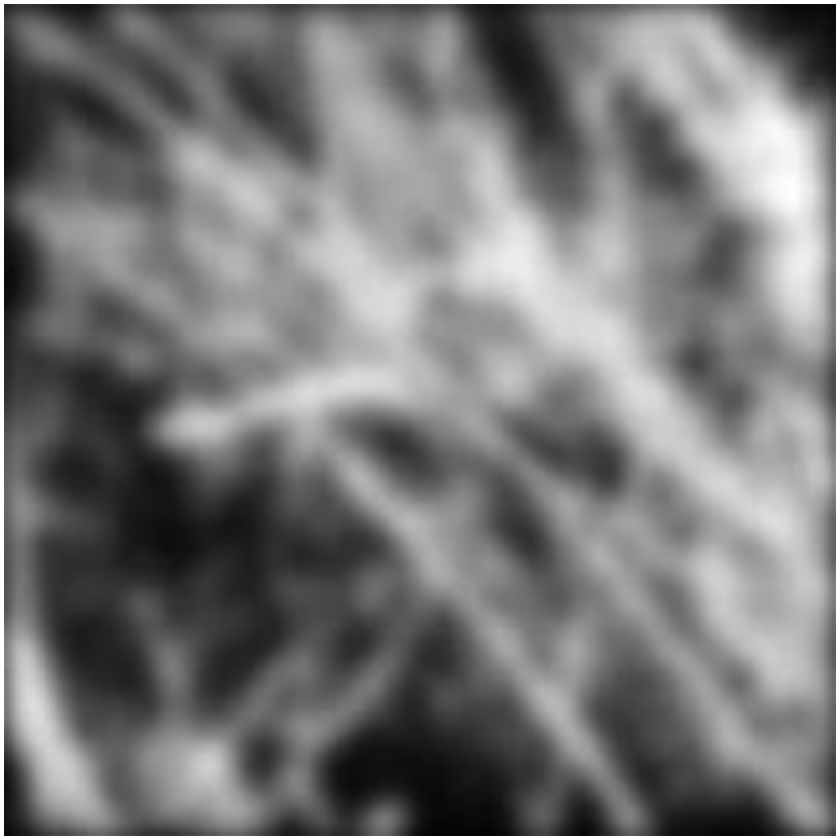




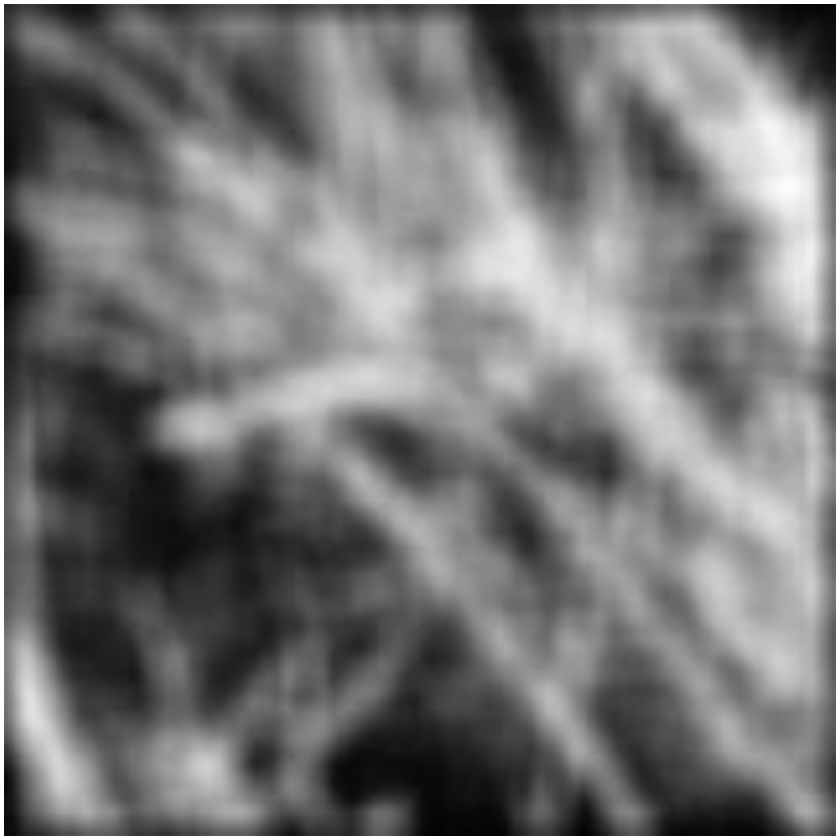
Difficulty in Spatial Domain

- Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

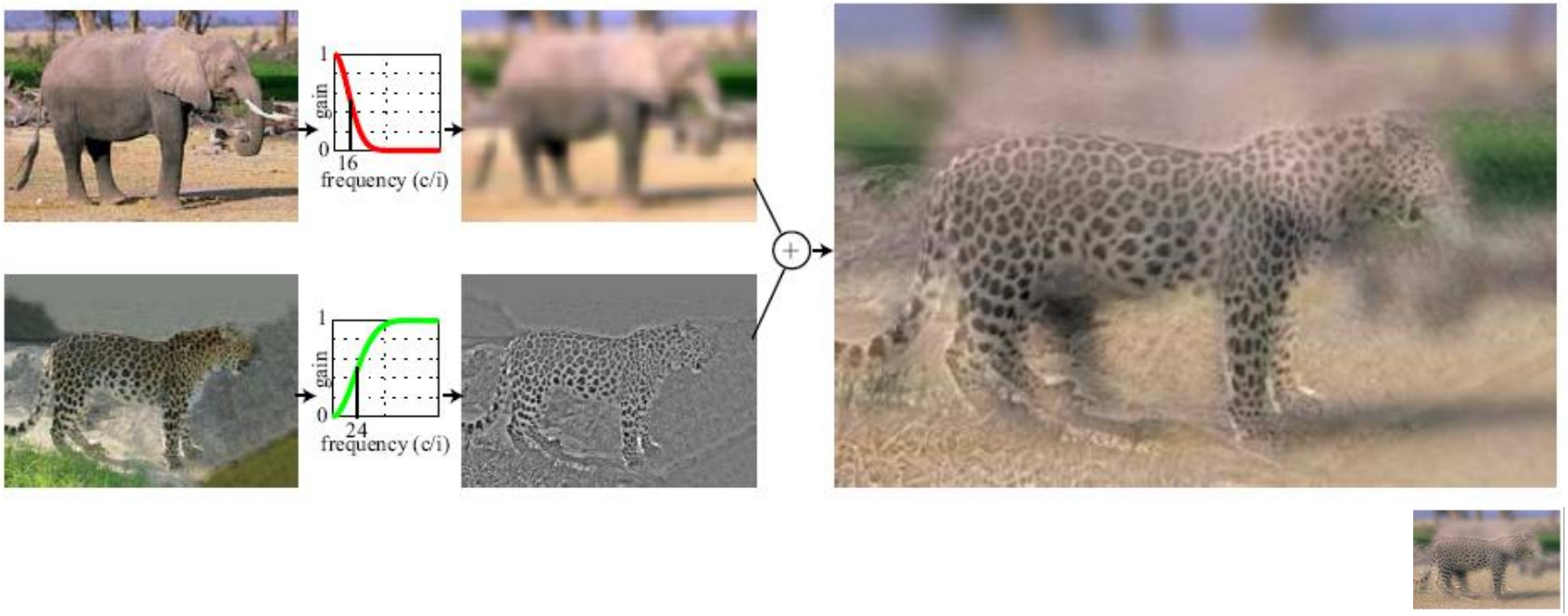
Gaussian 



Box filter 



Hybrid Images



- A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

Slides: Hoiem

Thinking in terms of frequency

Why does a lower resolution image still make sense to us? What do we lose?

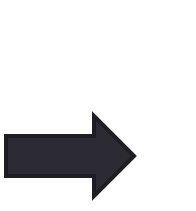


Image: http://www.flickr.com/photos/igorms/136916757/

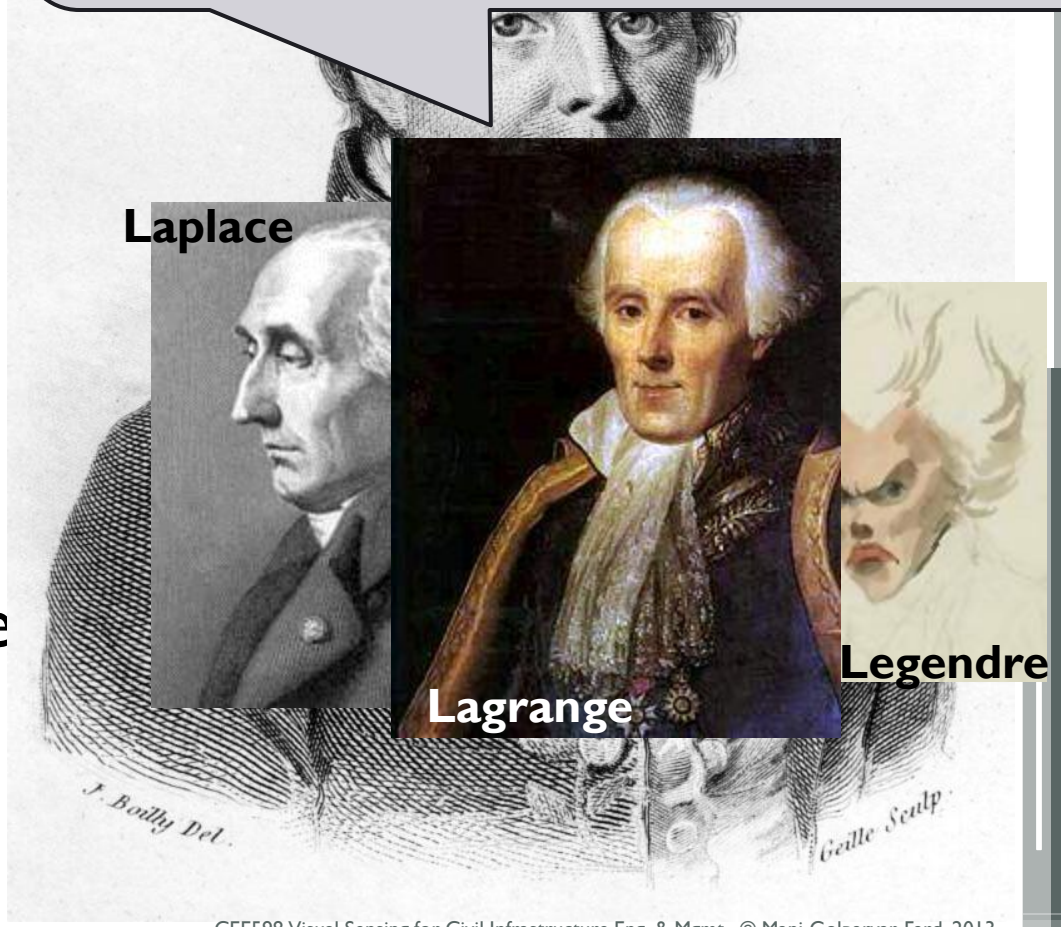
Jean Baptiste Joseph Fourier (1768-1830)

had crazy idea (1807):

Any *univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.*

- Don't believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it's (mostly) true
  - called Fourier Series
  - there are some subtle restrictions

...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.



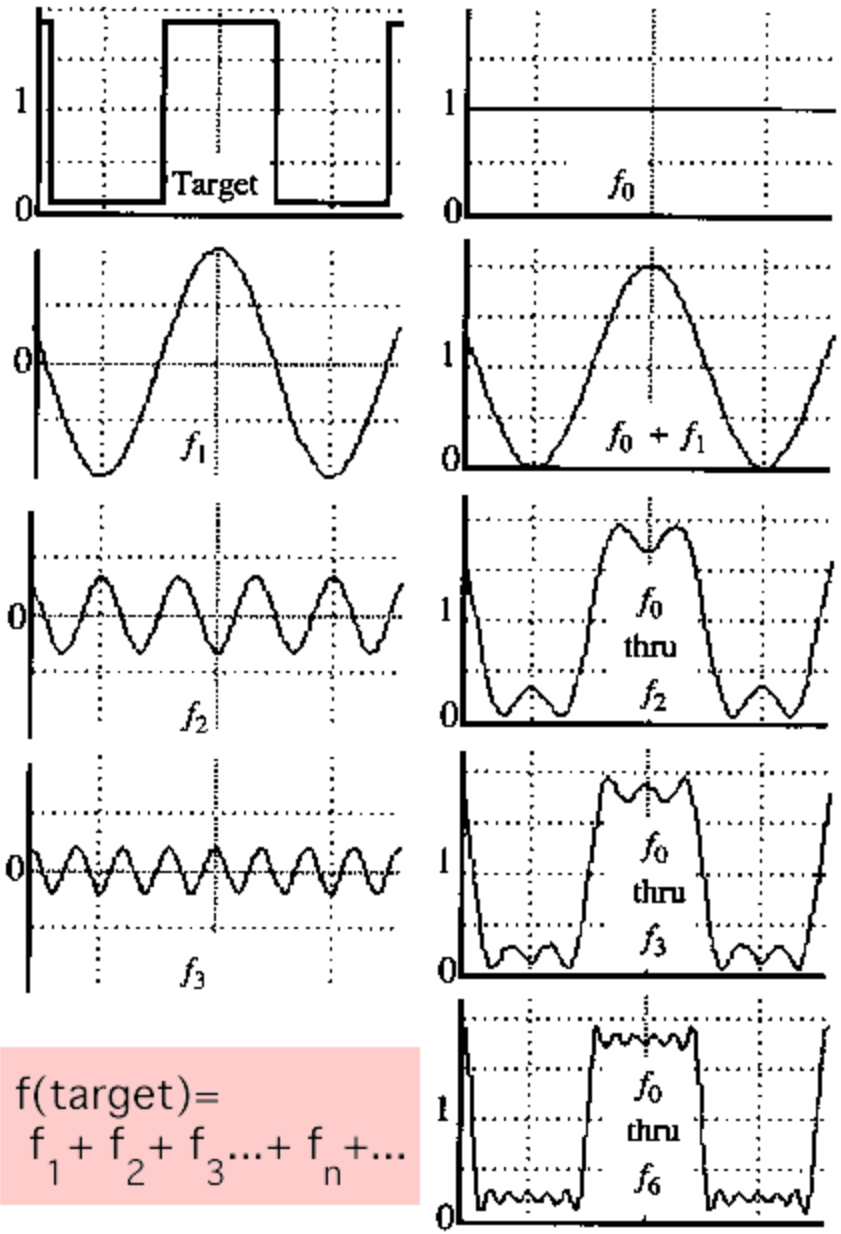
Slides: Hoiem

A sum of sines

Our building block:

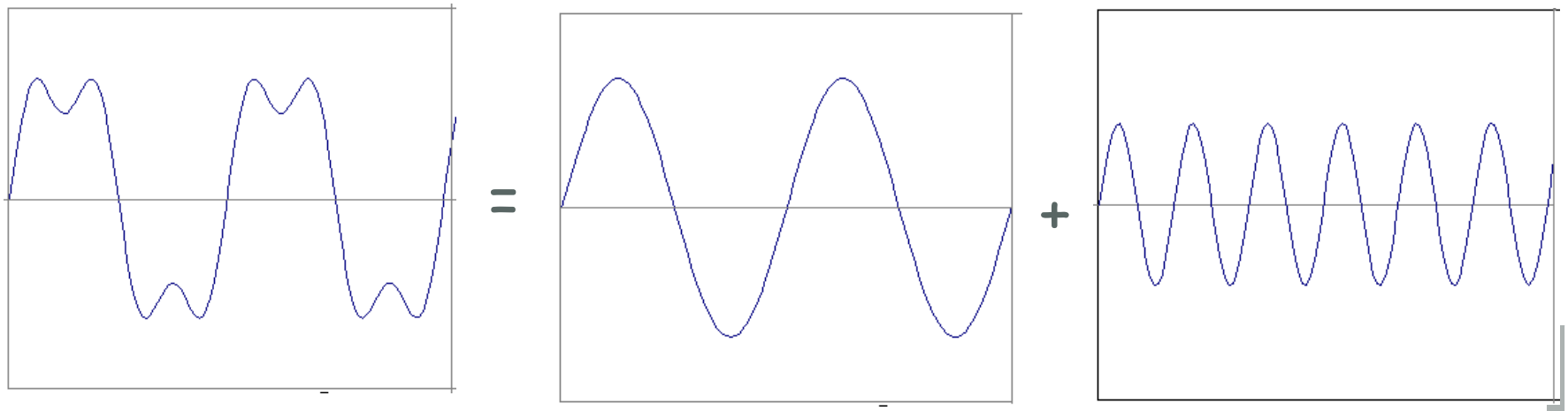
$$A\sin(\omega x + \phi)$$

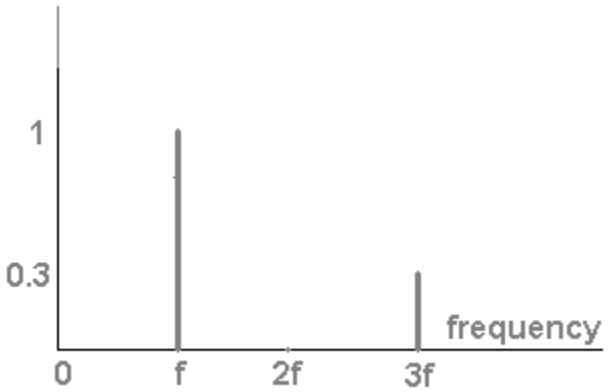
Add enough of them to get any signal *f(x)* you want!



Frequency Spectra

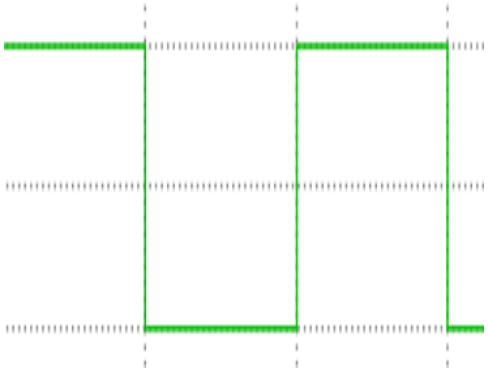
- example : $g(t) = \sin(2\pi f\, t) + (1/3)\sin(2\pi(3f)\, t)$



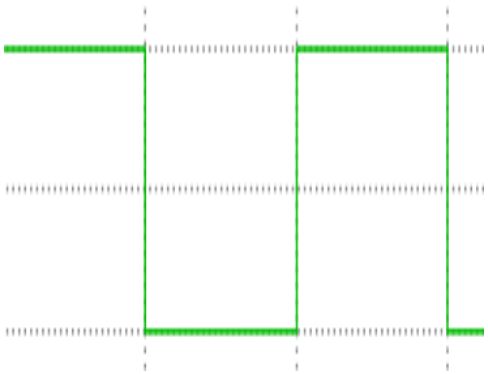


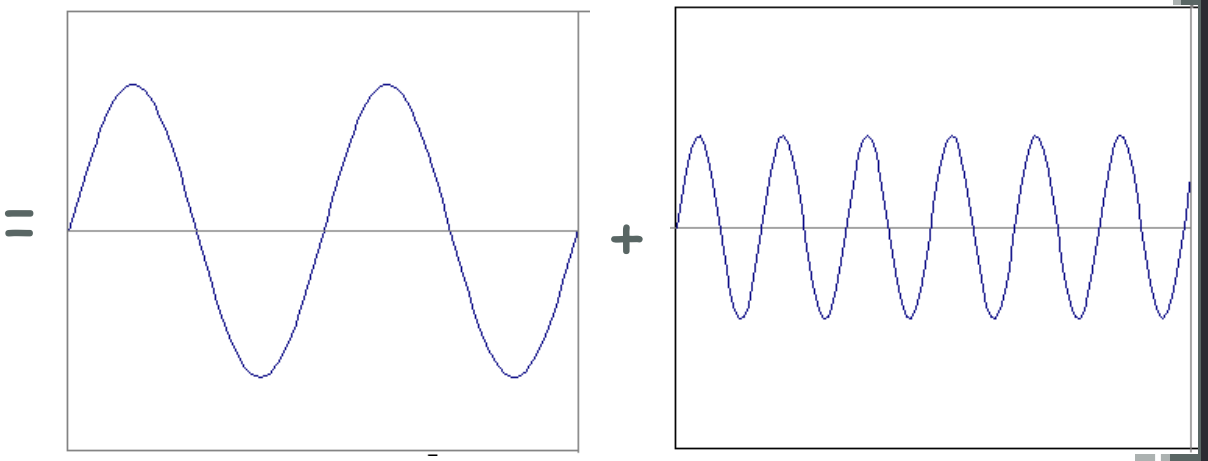
Slides: Efros

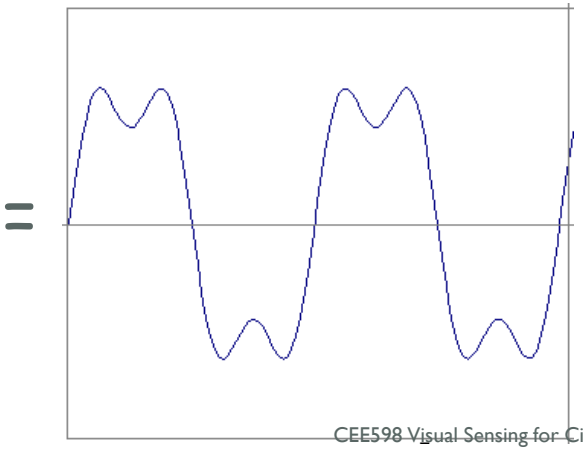
Frequency Spectra



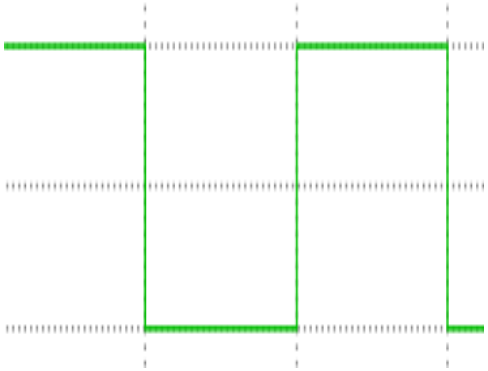
Frequency Spectra







Frequency Spectra



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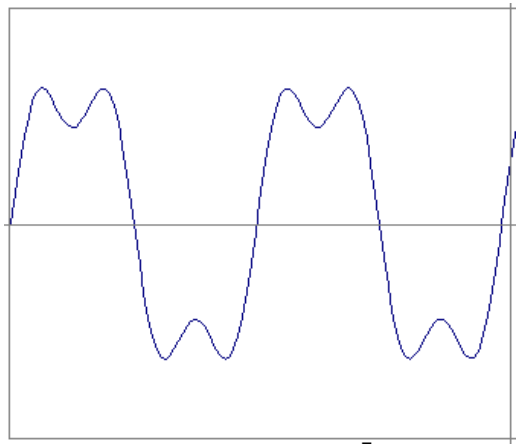

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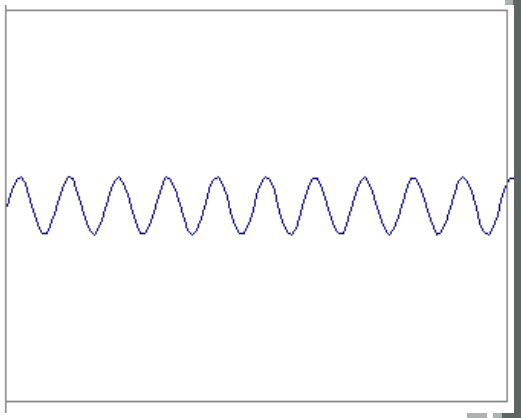

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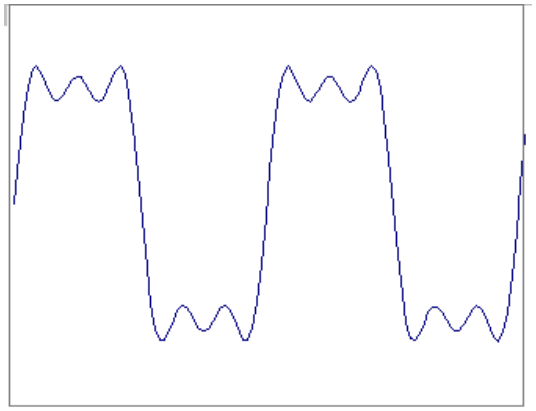
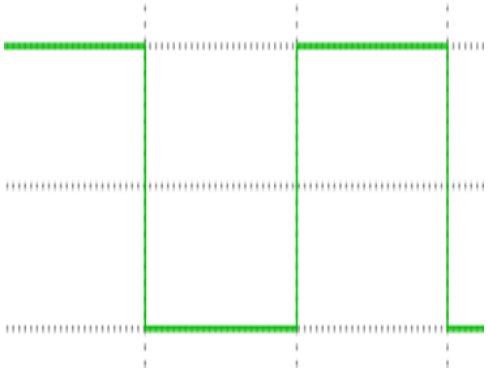
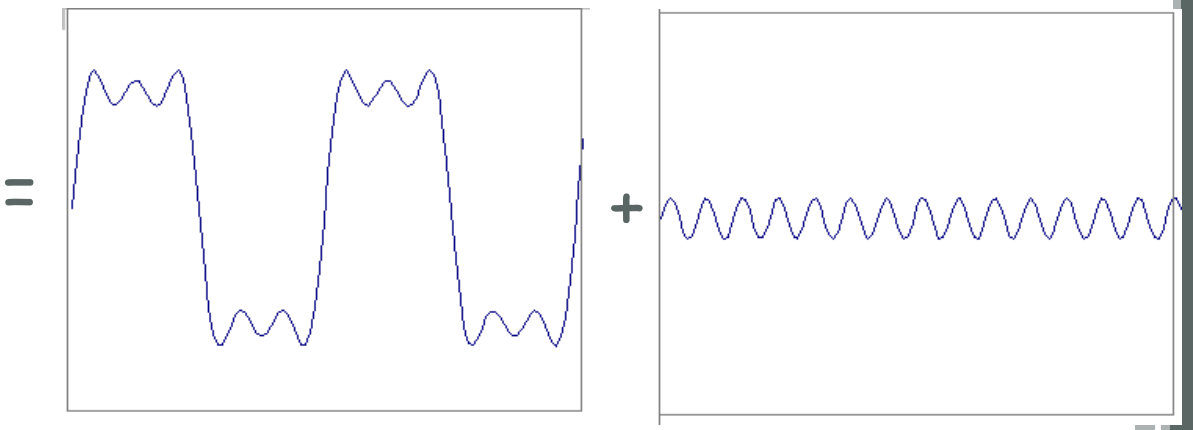
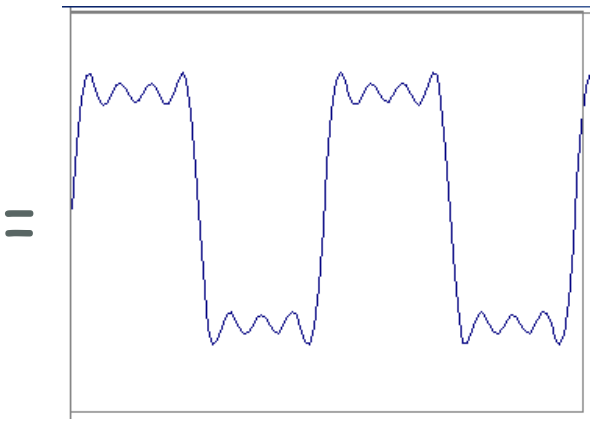


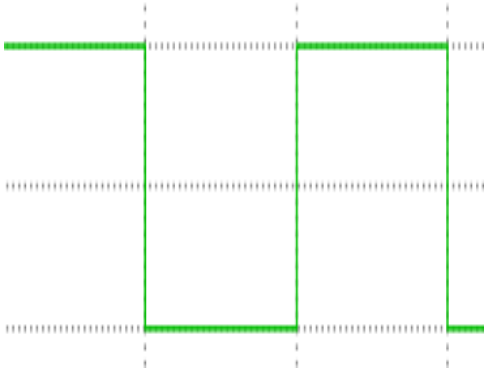
Frequency Spectra







Frequency Spectra



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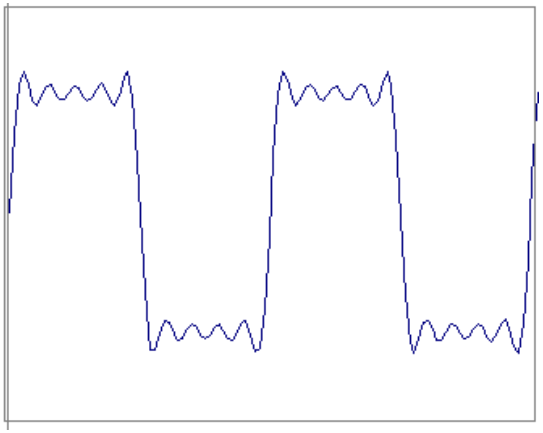

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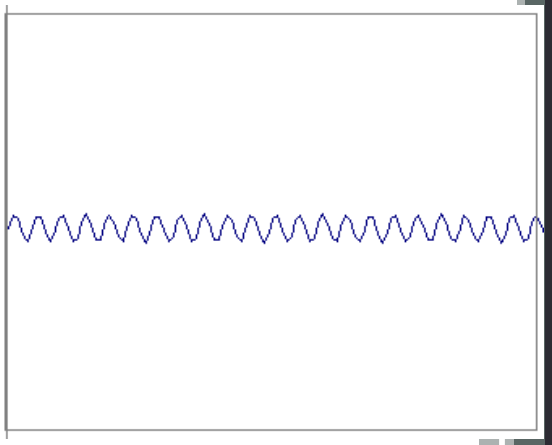

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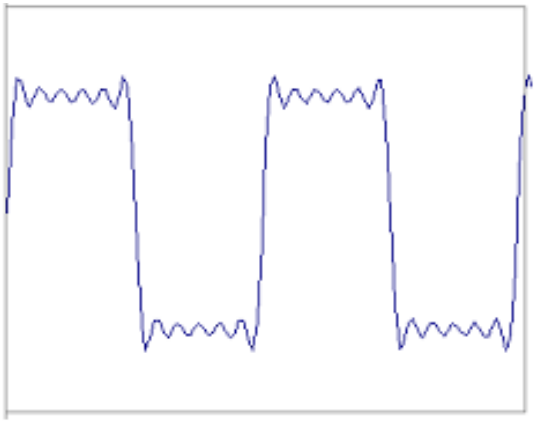
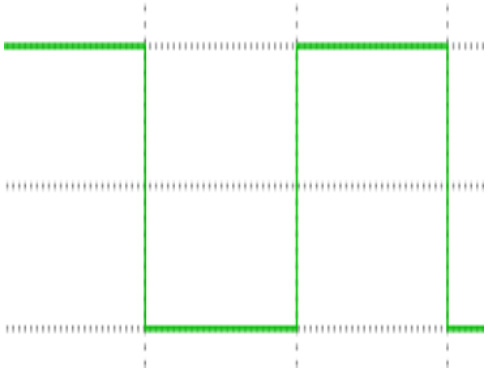
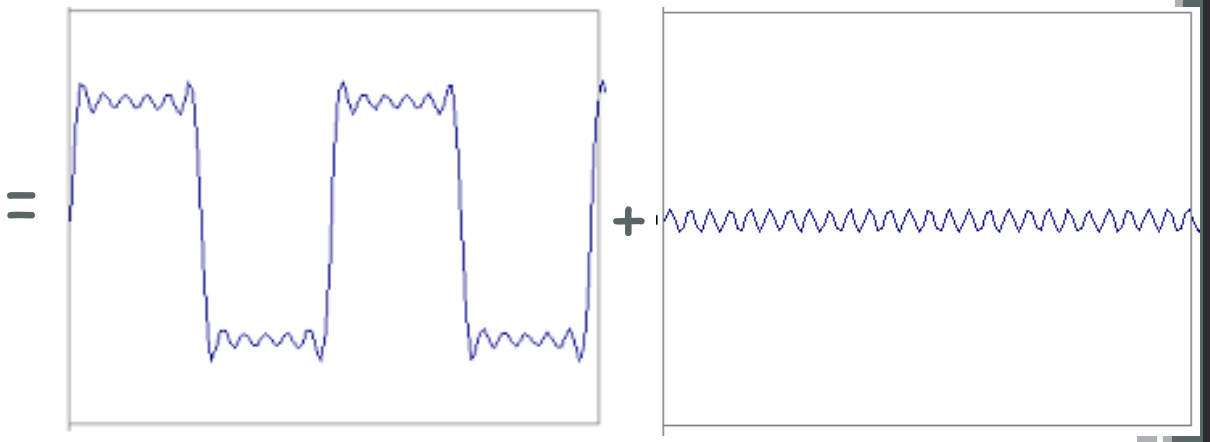
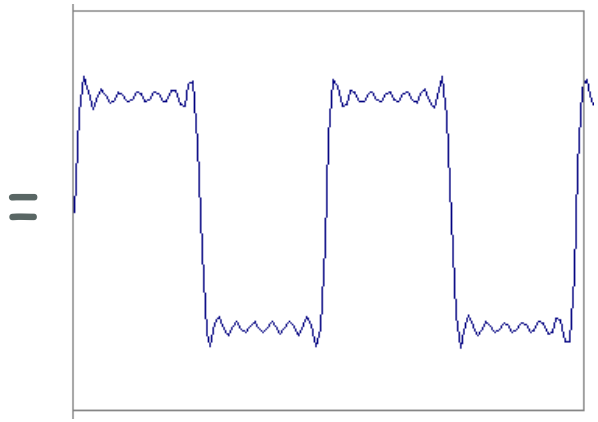


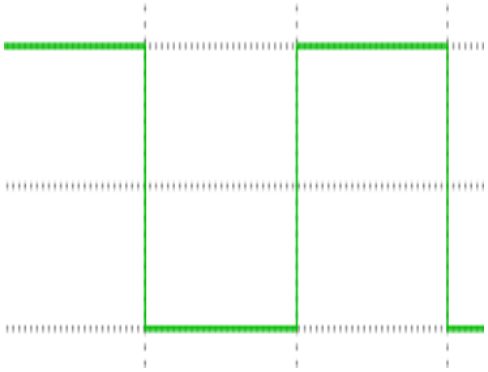
Frequency Spectra



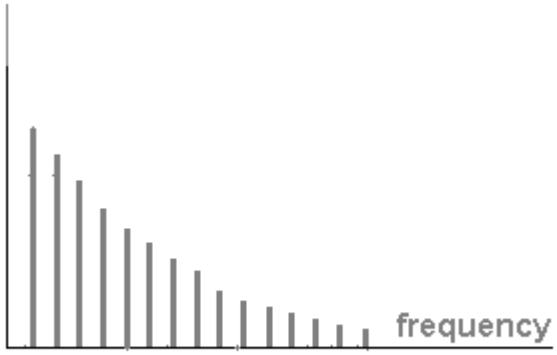




Frequency Spectra

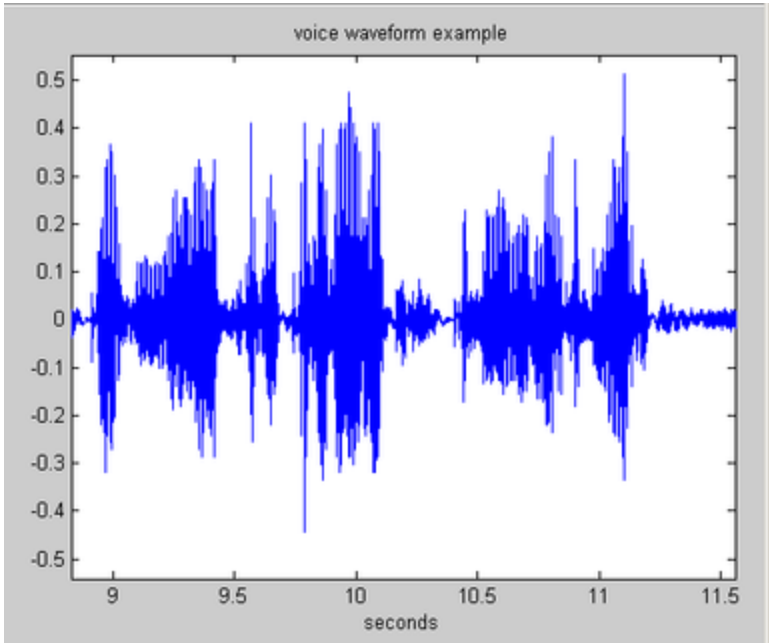


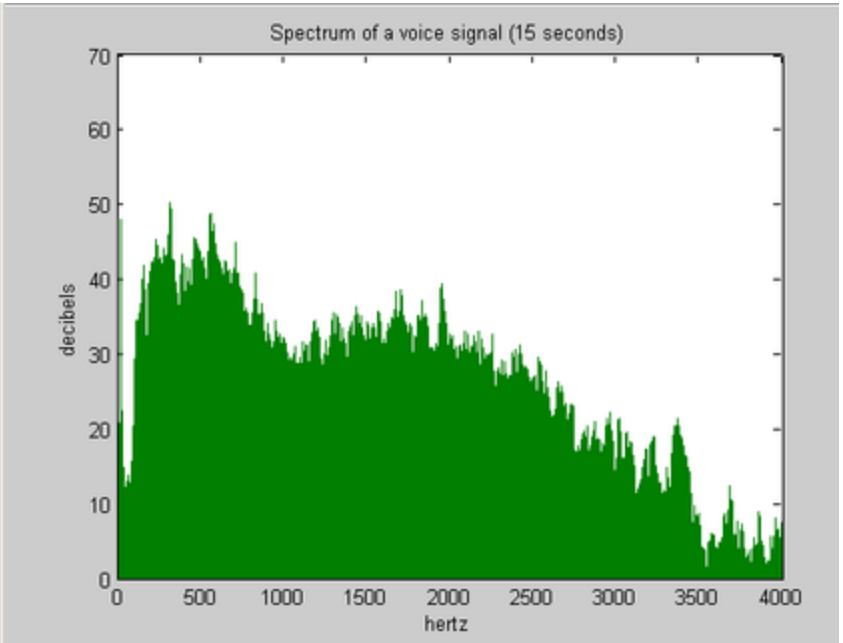
$$= A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)$$



Example: Music

- We think of music in terms of frequencies at different magnitudes

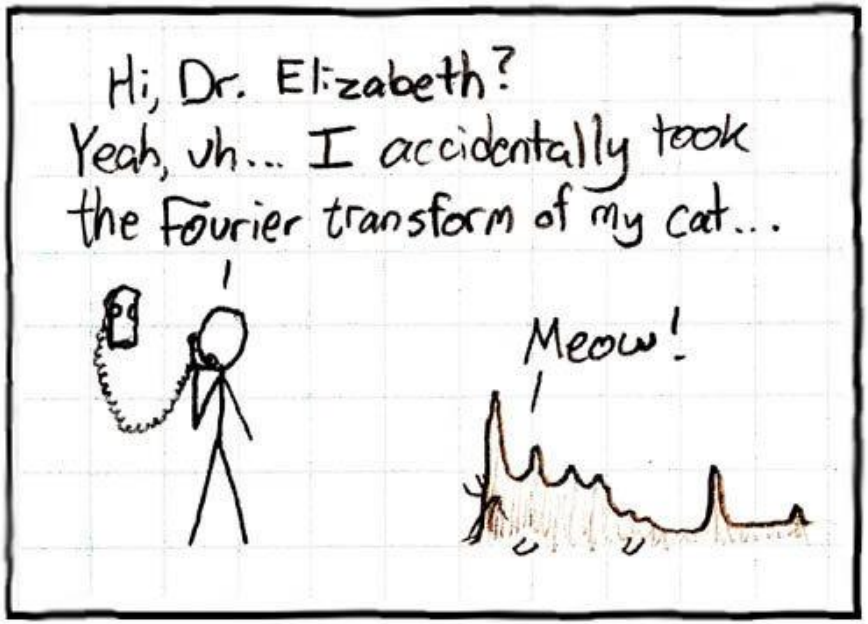




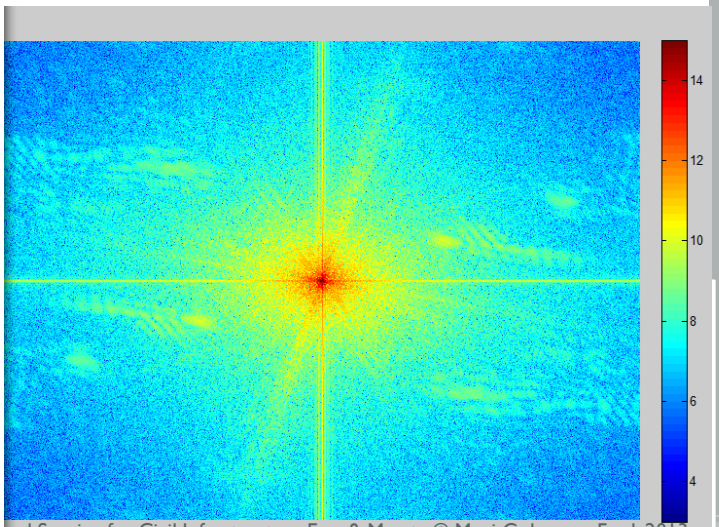
Other signals

- We can also think of all kinds of other signals the same way

Cats(?)



xkcd.com

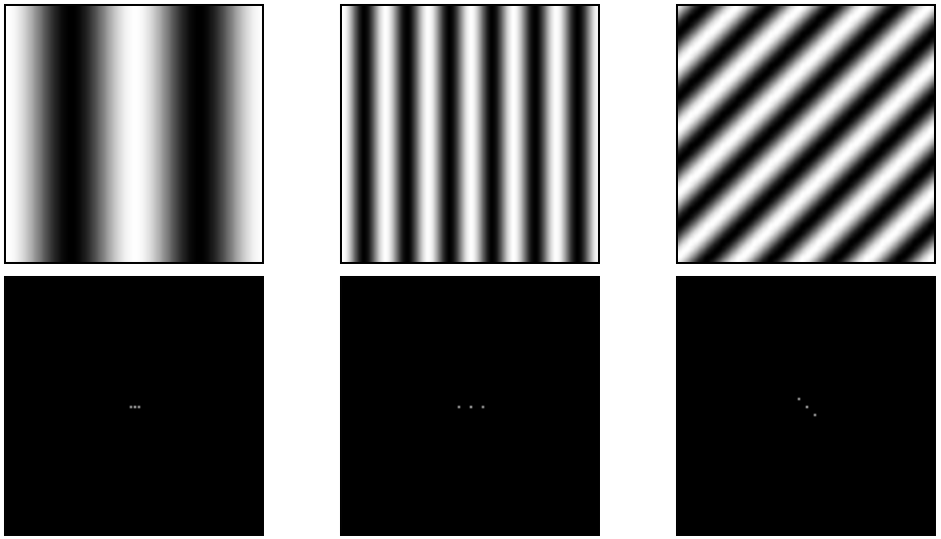


FFT of a cat

Fourier analysis in images

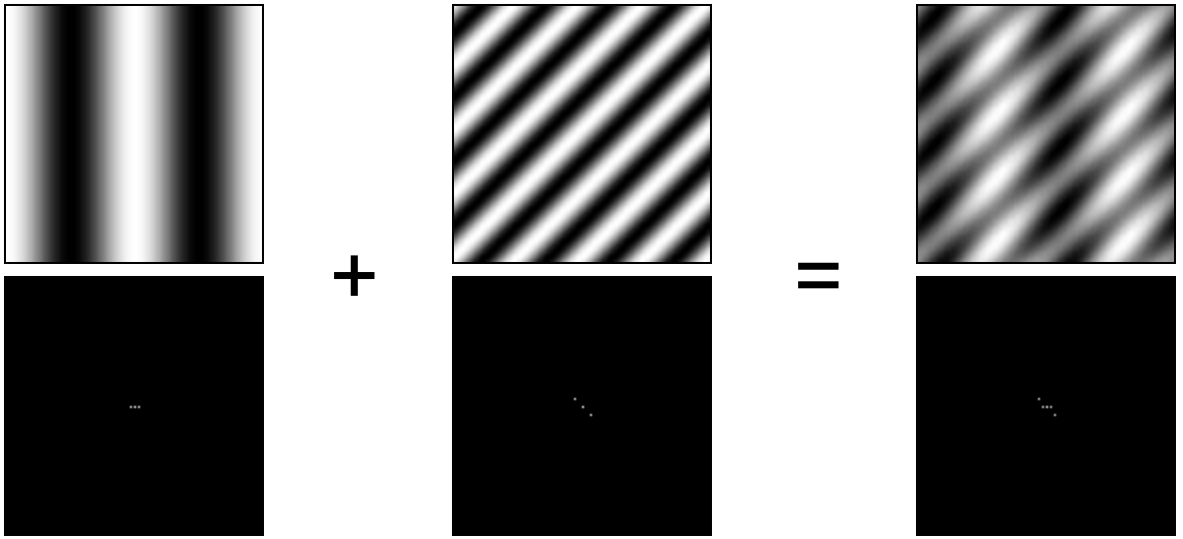
Intensity Image

Fourier Image

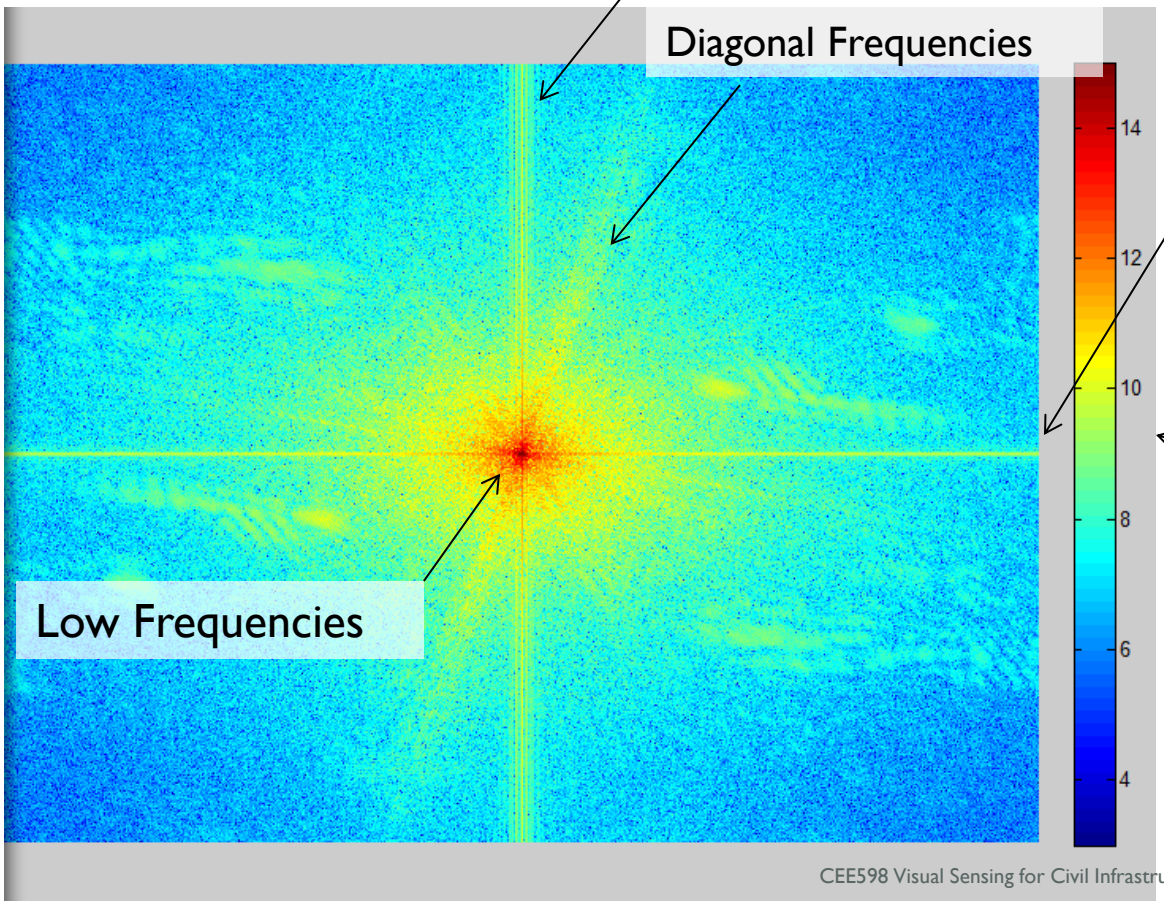


http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering

Signals can be composed



http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering
More: http://www.cs.unm.edu/~brayer/vision/fourier.html


Strong Vertical Frequency
(Sharp Horizontal Edge)
Diagonal Frequencies
Strong Horz. Frequency
(Sharp Vert. Edge)
Log Magnitude
Low Frequencies
14
12
10
8
6
4

Fourier Transform

- Fourier transform stores the magnitude and phase at each frequency
  - Magnitude encodes how much signal there is at a particular frequency
  - Phase encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of real and complex numbers

Amplitude: $A = \pm\sqrt{R(\omega)^2 + I(\omega)^2}$ Phase: $\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$

Euler's formula: $e^{inx} = \cos(nx) + i\sin(nx)$

Computing the Fourier Transform

$$H(\omega) = \mathcal{F}\{h(x)\} = Ae^{j\phi}$$

Continuous

$$H(\omega) = \int_{-\infty}^{\infty} h(x)e^{-j\omega x}dx$$

Discrete

$$H(k) = \frac{1}{N}\sum_{x=0}^{N-1} h(x)e^{-j\frac{2\pi kx}{N}}$$

k = -*N*/2..*N*/2

Fast Fourier Transform (FFT): NlogN

The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$\mathrm{F}[g * h] = \mathrm{F}[g]\mathrm{F}[h]$$

- The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$\mathrm{F}^{-1}[gh] = \mathrm{F}^{-1}[g] * \mathrm{F}^{-1}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

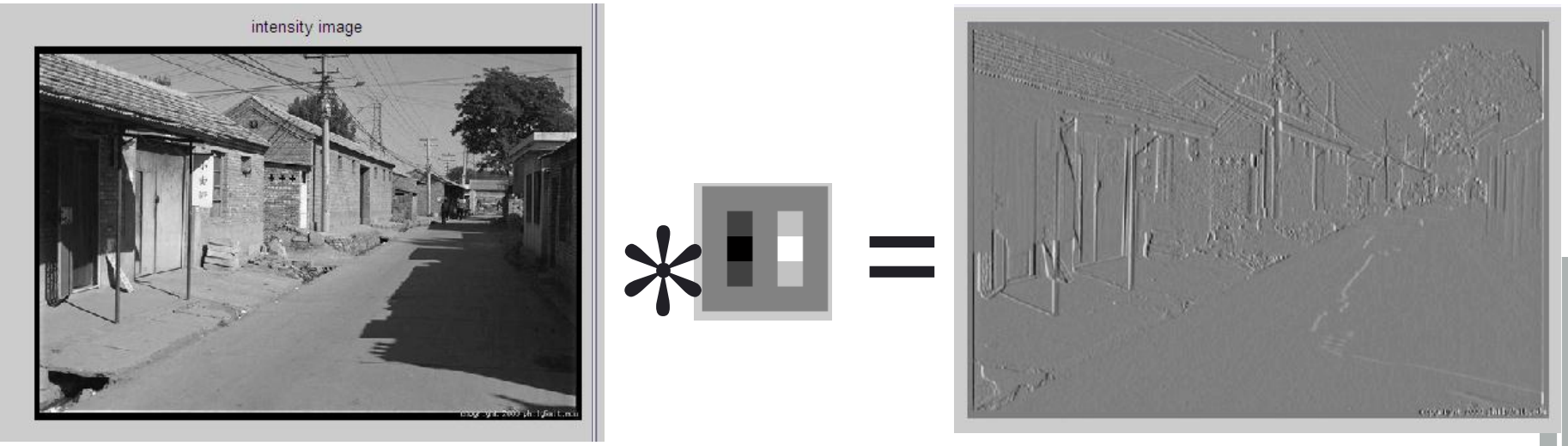
Properties of Fourier Transforms

- Linearity $\mathcal{F}[ax(t) + by(t)] = a\mathcal{F}[x(t)] + b\mathcal{F}[y(t)]$
- Fourier transform of a real signal is symmetric about the origin
- The energy of the signal is the same as the energy of its Fourier transform

See Szeliski Book (3.4)

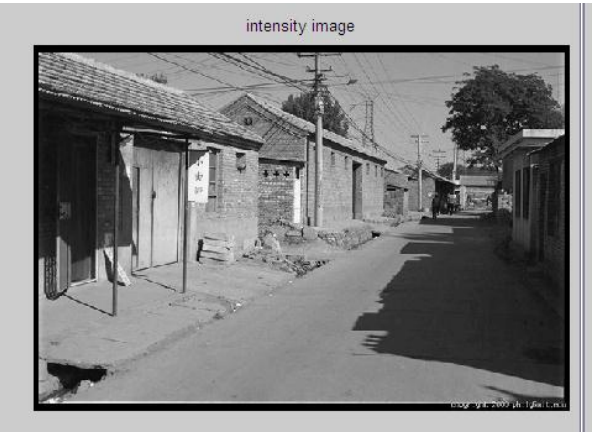
Filtering in spatial domain

1	0	-1
2	0	-2
1	0	-1

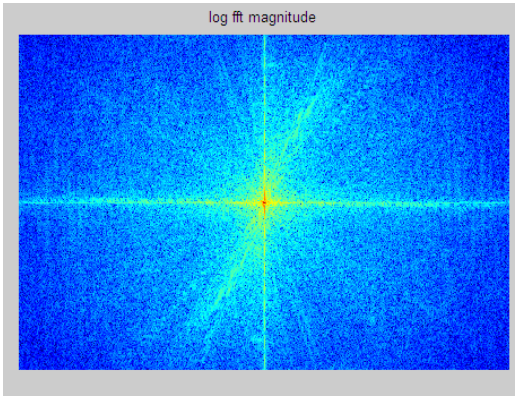


Filtering in frequency domain

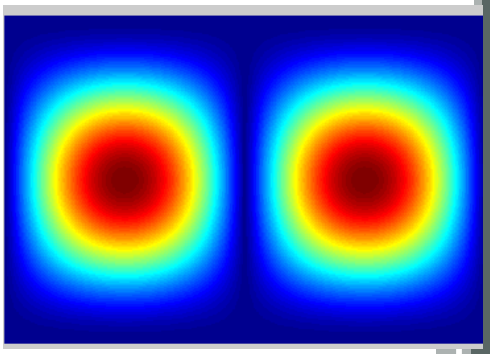
FFT



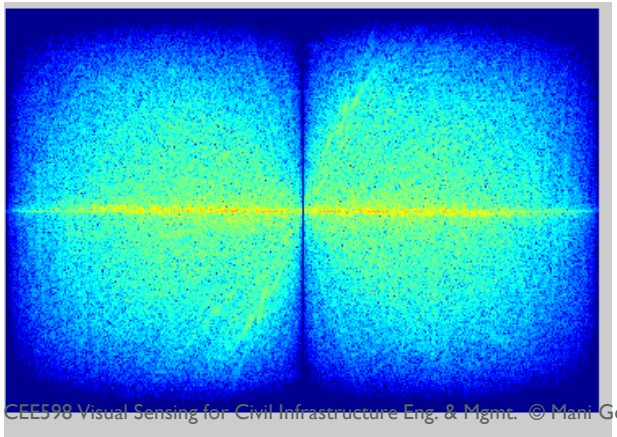
FFT



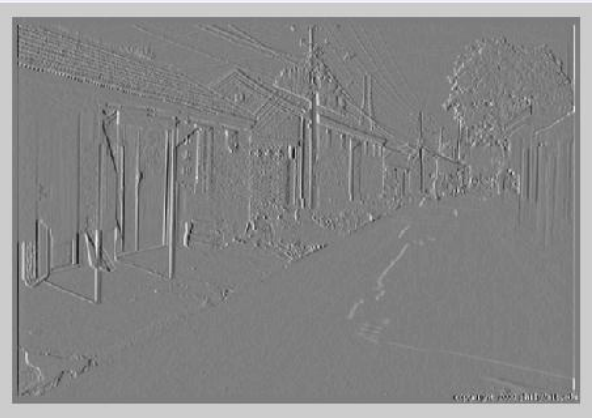
×



=



Inverse FFT



Fourier Matlab demo

FFT in Matlab

- Filtering with fft

```
im = ... % "im" should be a gray-scale floating point image
[imh, imw] = size(im);
fftsize = 1024; % should be order of 2 (for speed) and include padding
im_fft = fft2(im, fftsize, fftsize); % 1) fft im with padding
hs = 50; % filter half-size
fil = fspecial('gaussian', hs*2+1, 10);
fil_fft = fft2(fil, fftsize, fftsize); % 2) fft fil, pad to same size as image
im_fil_fft = im_fft .* fil_fft; % 3) multiply fft images
im_fil = ifft2(im_fil_fft); % 4) inverse fft2
im_fil = im_fil(1+hs:size(im,1)+hs, 1+hs:size(im, 2)+hs); % 5) remove padding
```

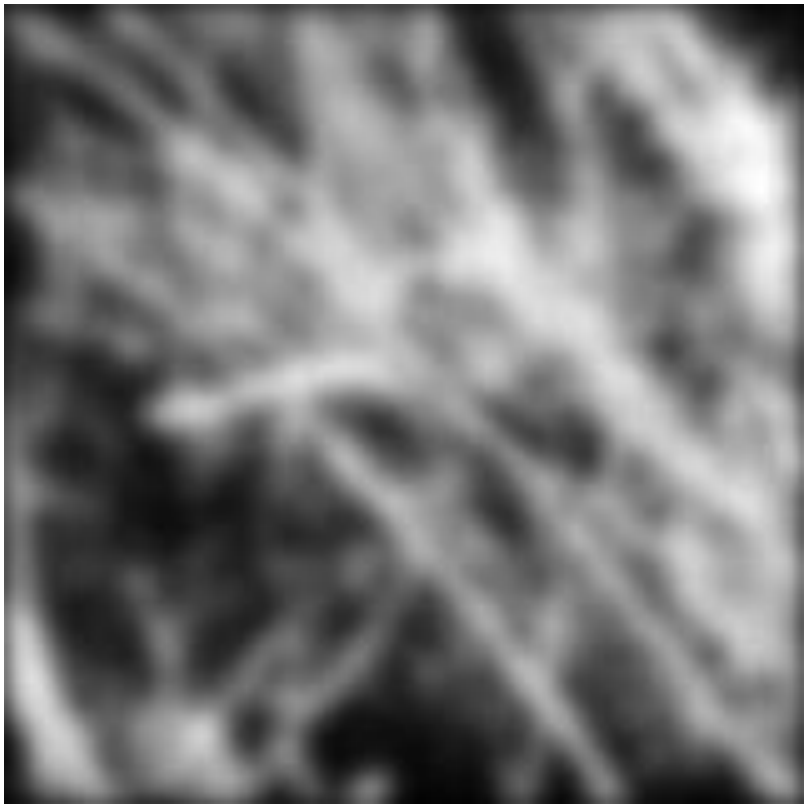
- Displaying with fft

```
figure(1), imagesc(log(abs(fftshift(im_fft)))), axis image, colormap jet
```

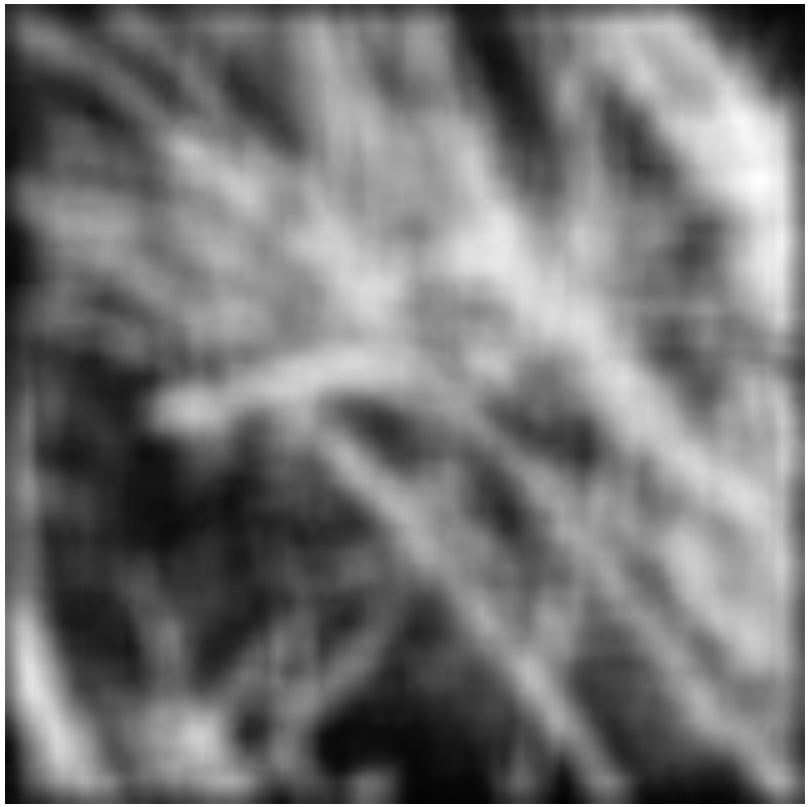
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

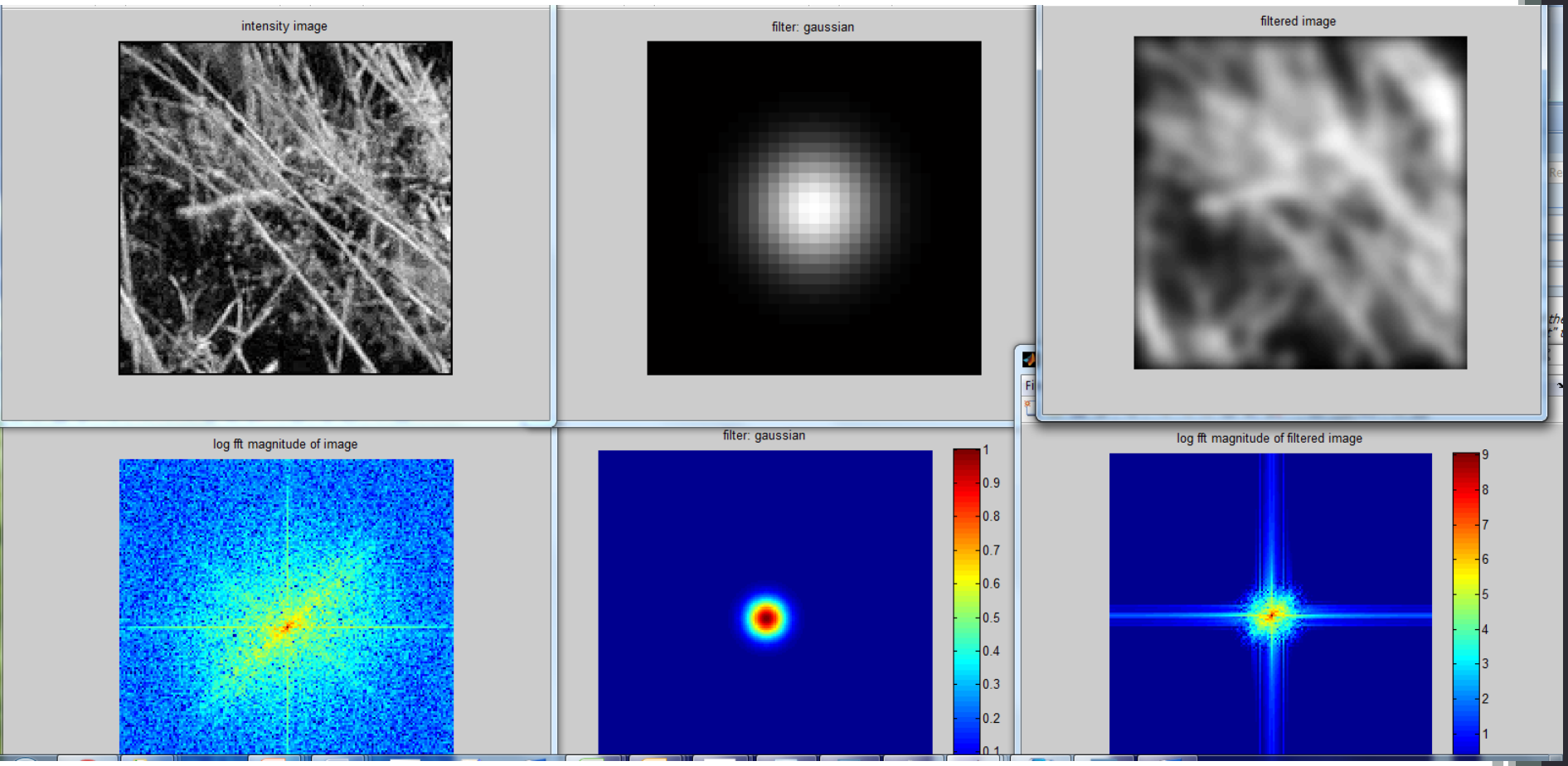
Gaussian 



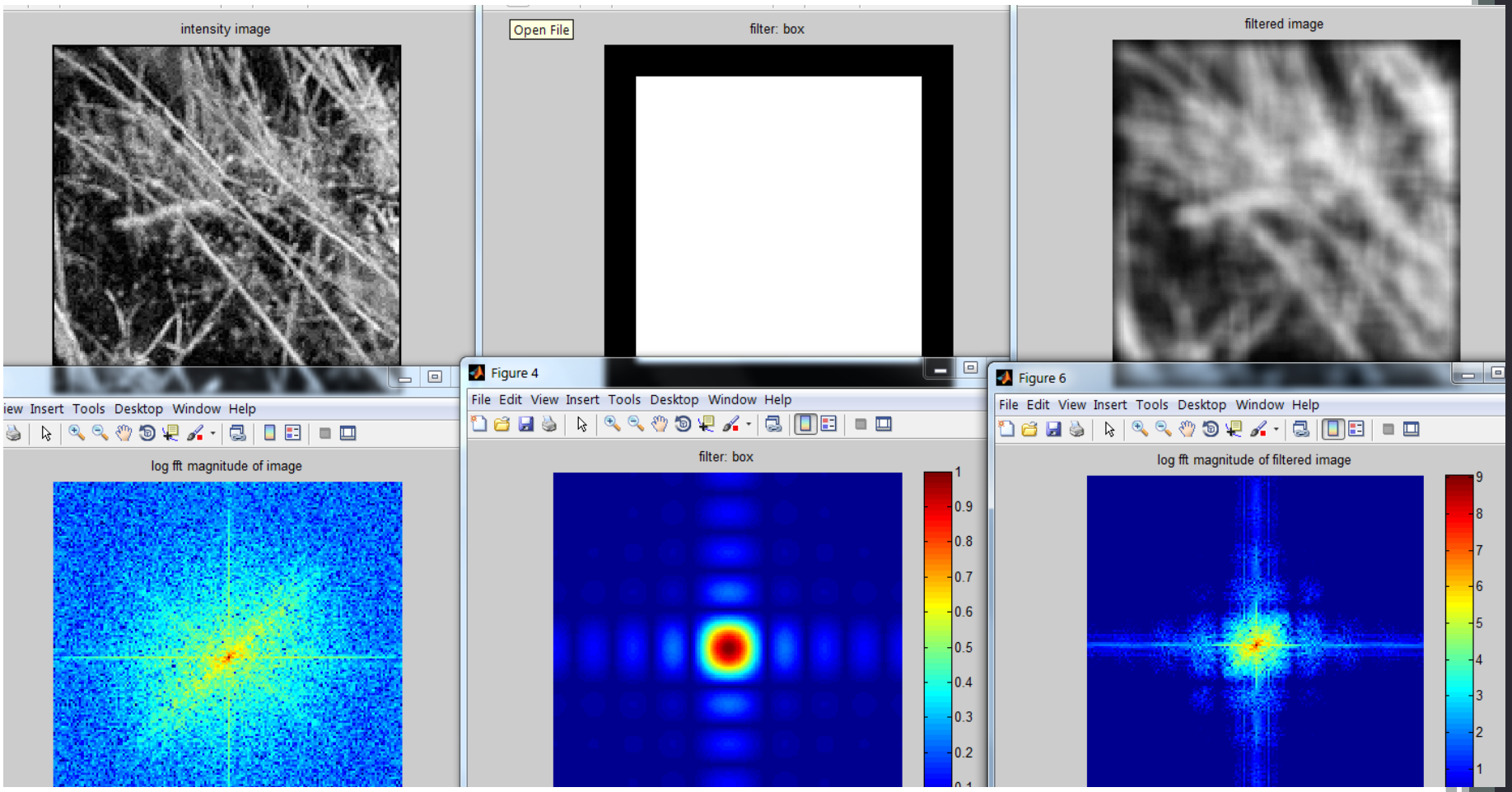
Box filter 



Gaussian



Box Filter

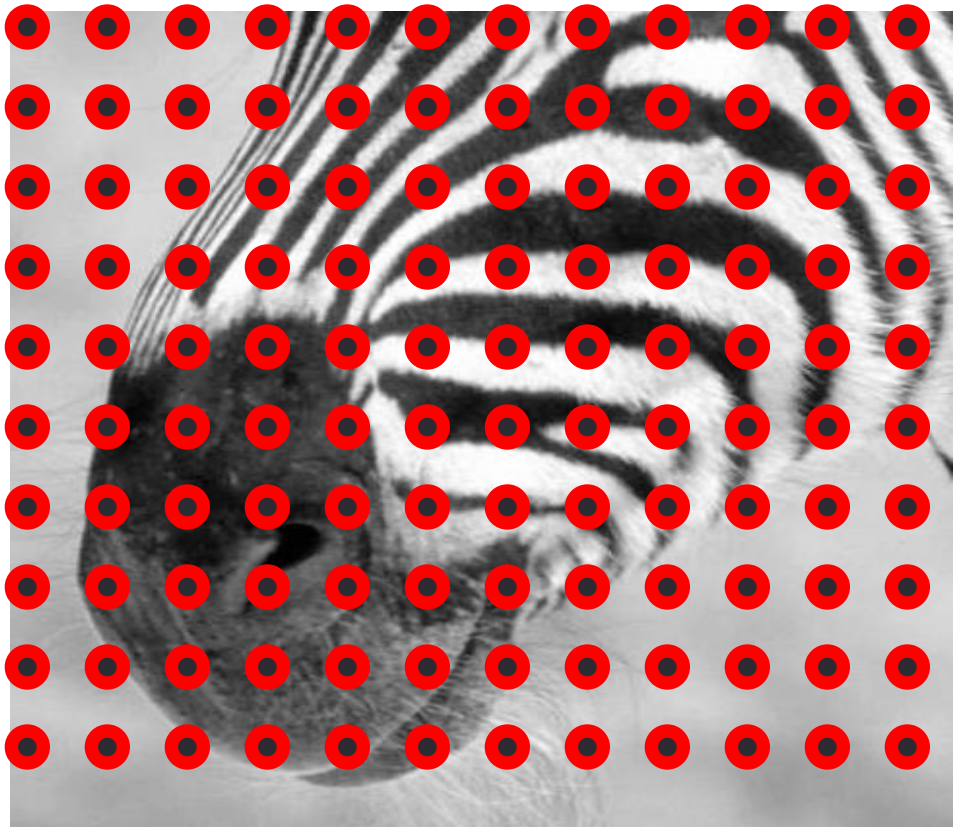

intensity image
Open File
filter: box
filtered image
iew Insert Tools Desktop Window Help
log fft magnitude of image
Figure 4
File Edit View Insert Tools Desktop Window Help
filter: box
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
Figure 6
File Edit View Insert Tools Desktop Window Help
log fft magnitude of filtered image
9
8
7
6
5
4
3
2
1

Sampling

Why does a lower resolution image still make sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

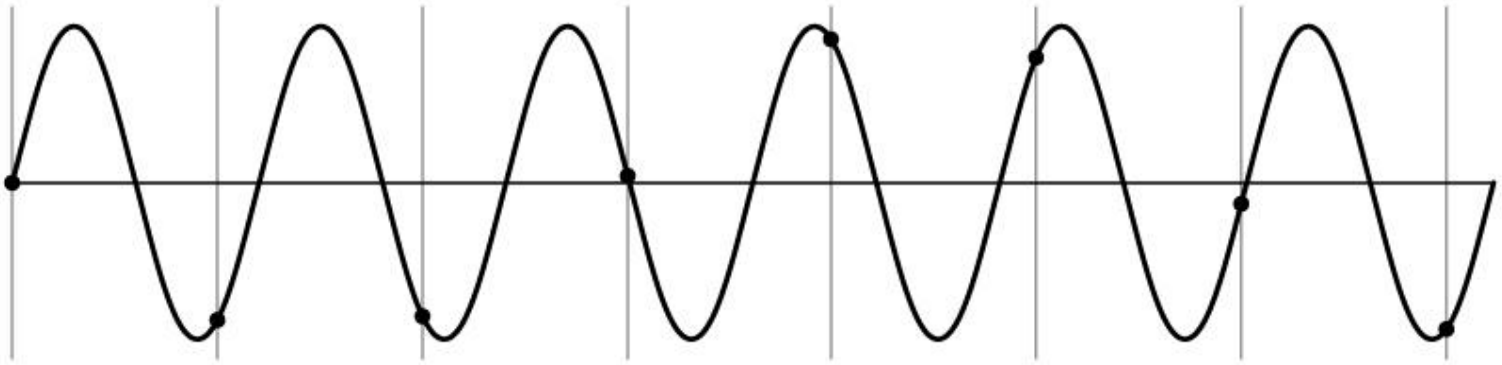
Subsampling by a factor of 2



Throw away every other row and column to create a 1/2 size image

Aliasing problem

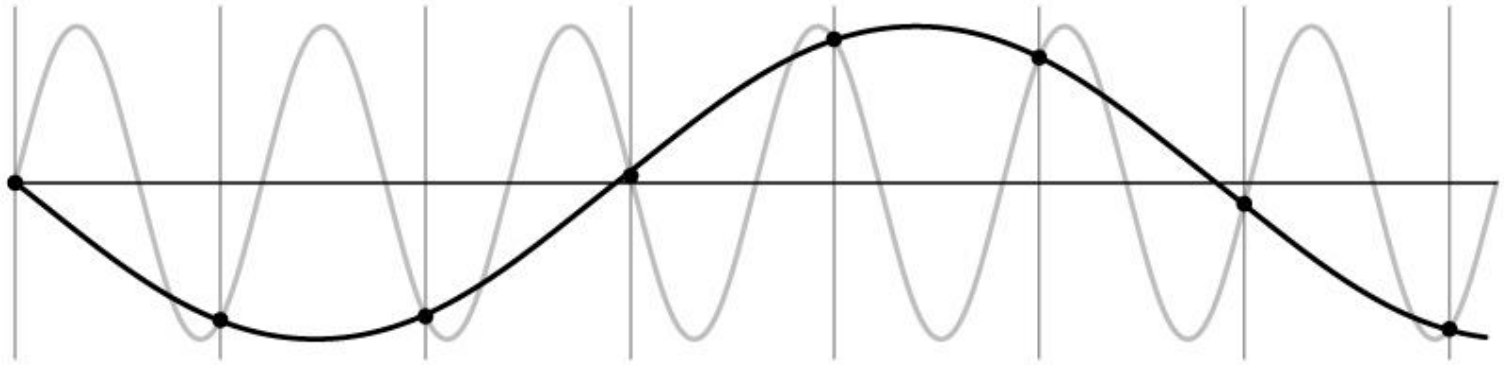
- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

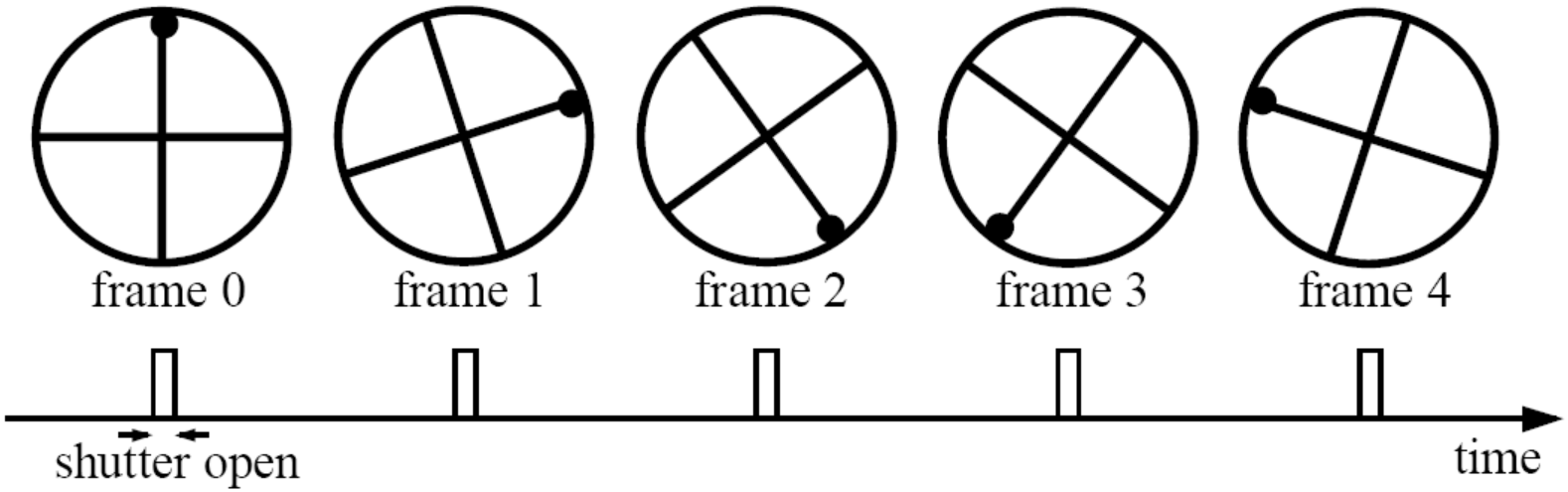
- Sub-sampling may be dangerous….
- Characteristic errors may appear:
  - "Wagon wheels rolling the wrong way in movies"
  - "Checkerboards disintegrate in ray tracing"
  - "Striped shirts look funny on color television"

Source: D. Forsyth

Aliasing in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

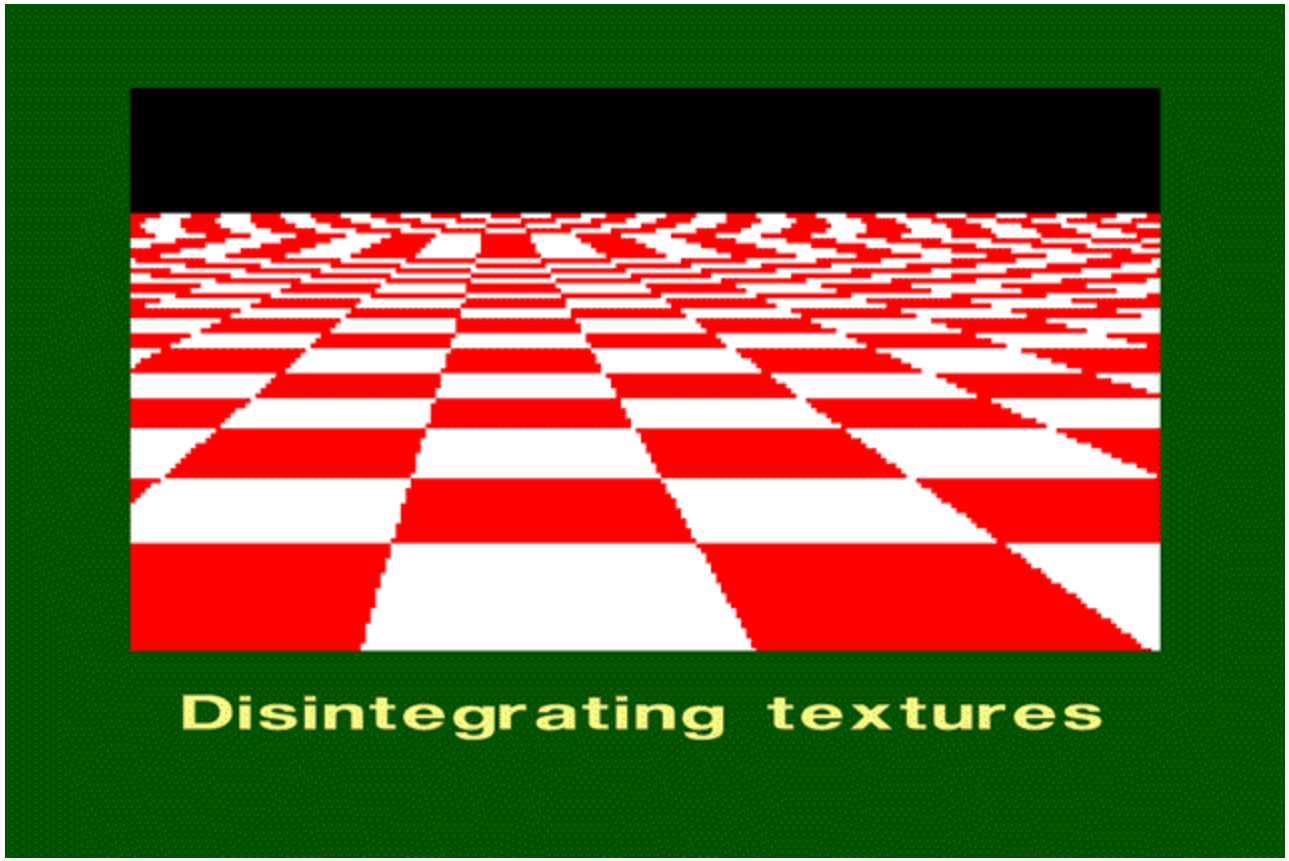
If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

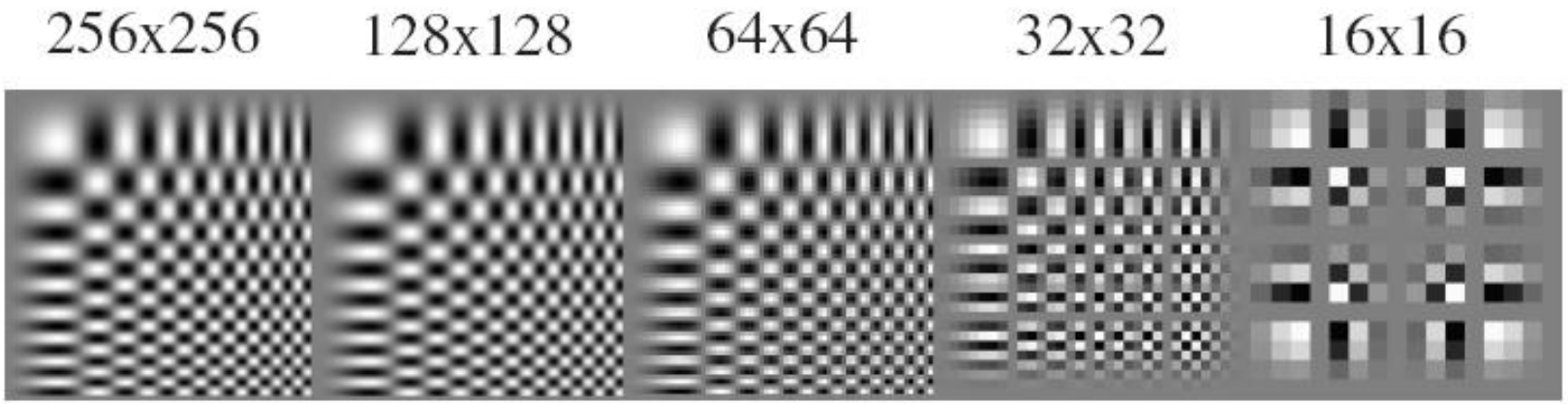
Slide by Steve Seitz

Aliasing in graphics



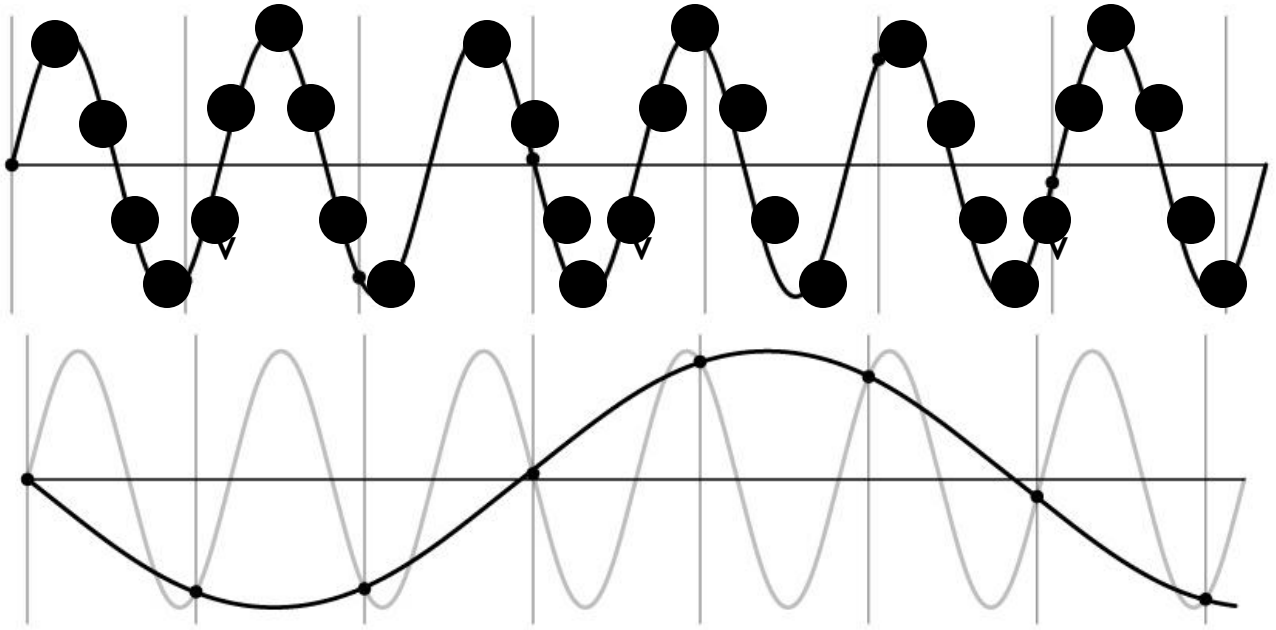
Source: A. Efros

Sampling and aliasing



Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{max}$
- f_{max} = max frequency of the input signal
- This will allows to reconstruct the original perfectly from the sampled version



Anti-aliasing

Solutions:

- Sample more often

- Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it's better than aliasing
  - Apply a smoothing filter

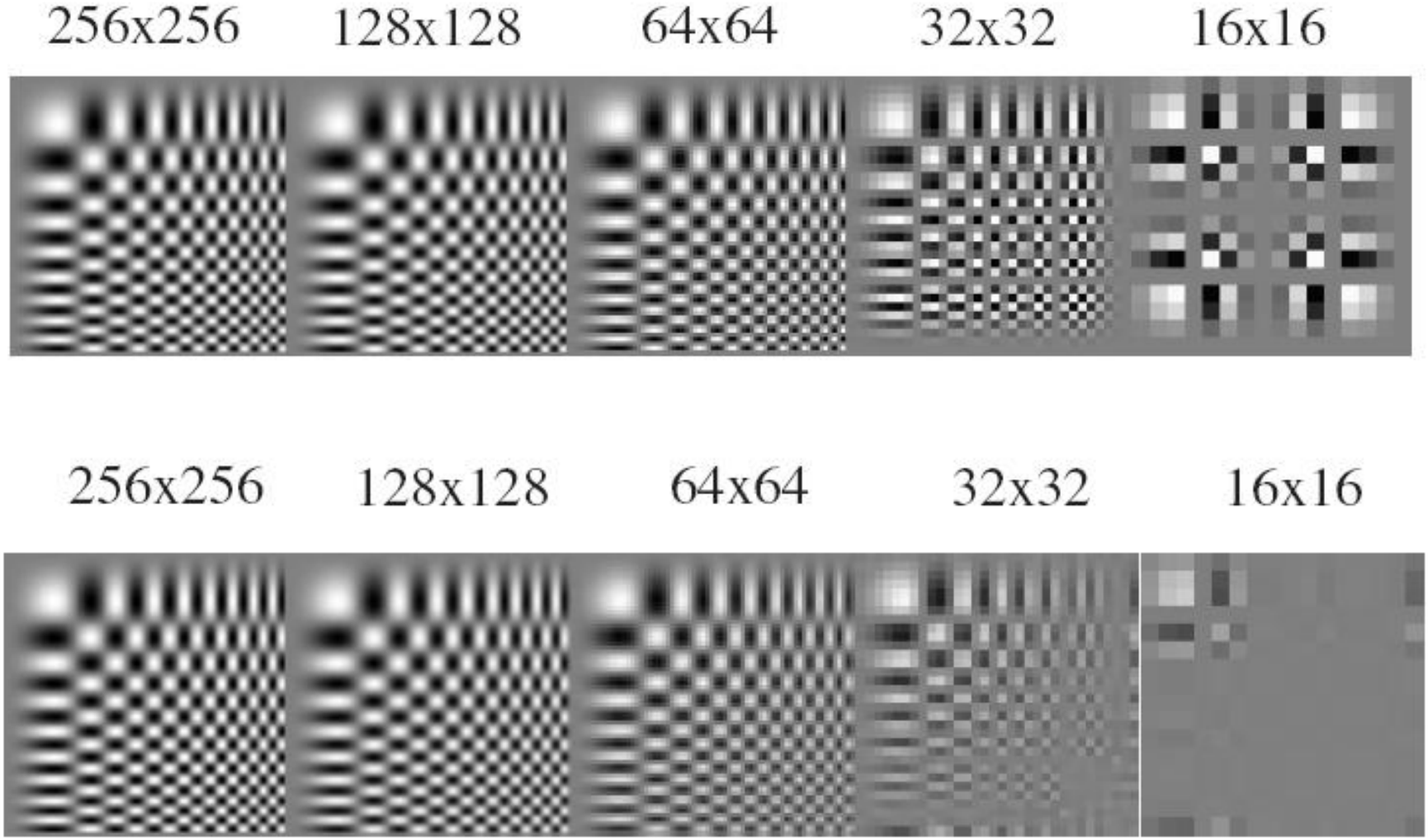
Algorithm for downsampling by factor of 2

1. Start with image(h, w)
2. Apply low-pass filter

   im_blur = imfilter(image, fspecial('gaussian', 7, 1))
3. Sample every other pixel

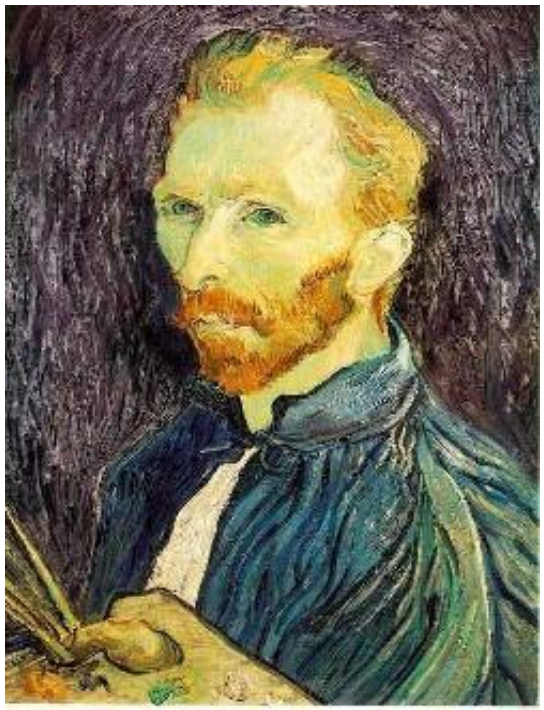
   im_small = im_blur(1:2:end, 1:2:end);

Anti-aliasing



Forsyth and Ponce 2002

Subsampling without pre-filtering



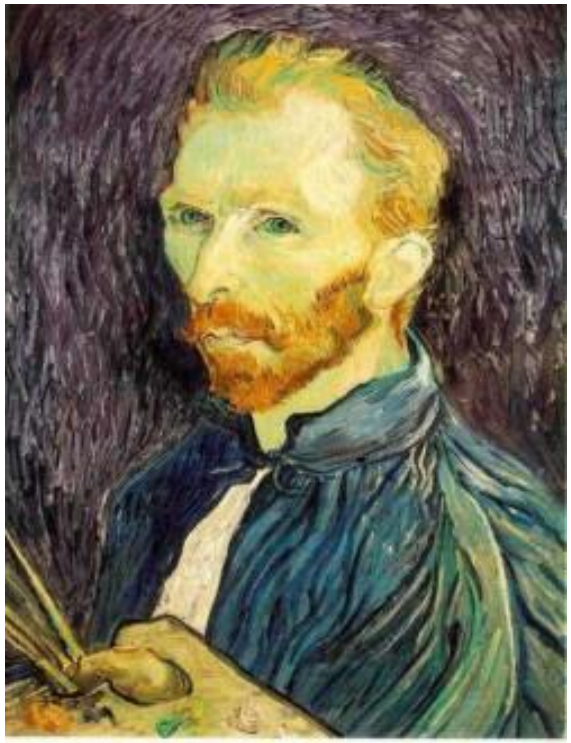
1/2

1/4 (2x zoom)

1/8 (4x zoom)

Slide by Steve Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

G 1/8

Slide by Steve Seitz

Why does a lower resolution image still make sense to us? What do we lose?

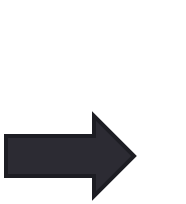
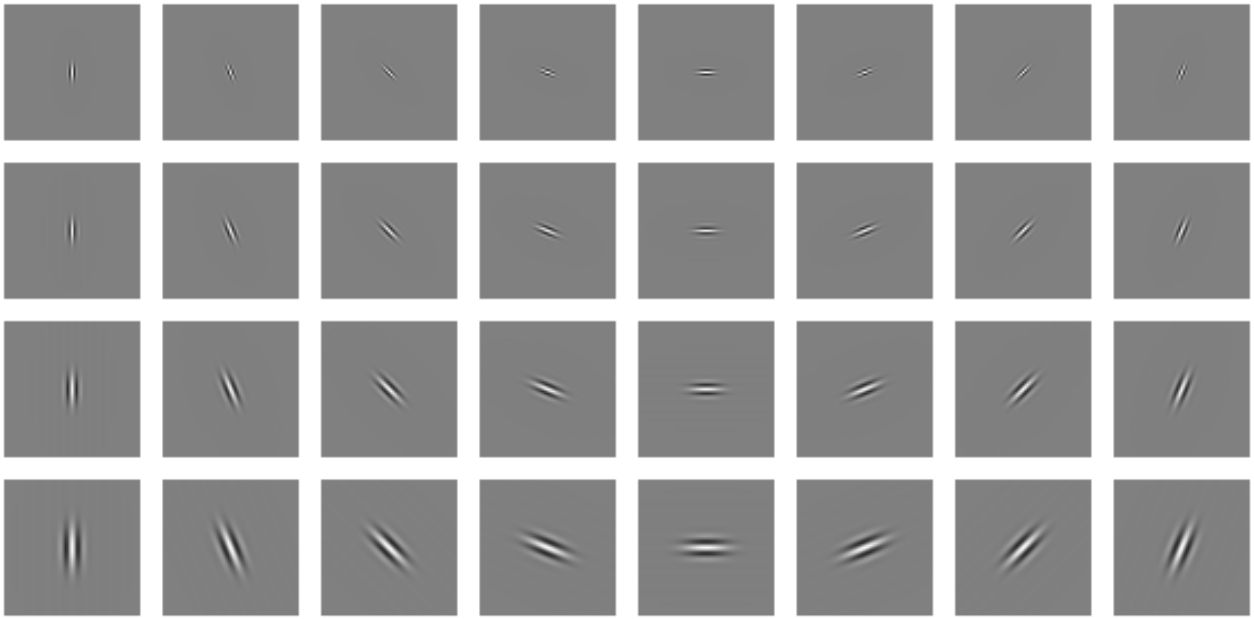


Image: http://www.flickr.com/photos/igorms/136916757/

Clues from Human Perception

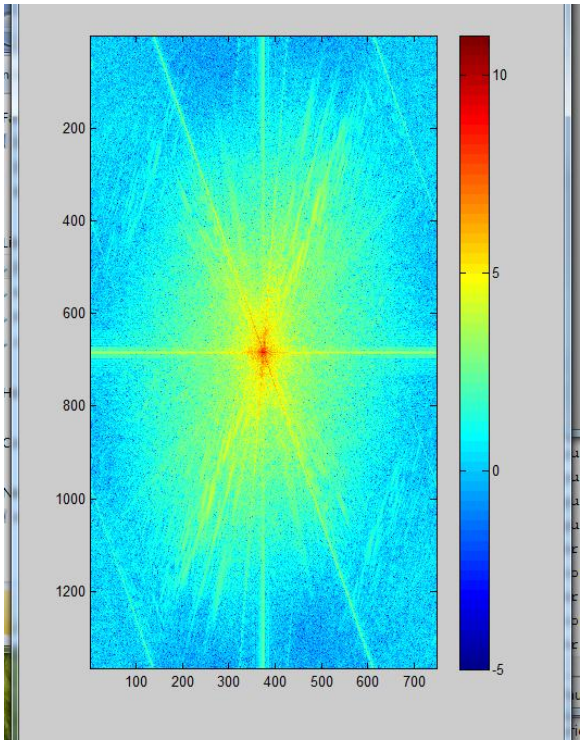
- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid-high frequencies dominate perception
- When we see an image from far away, we are effectively subsampling it



Early Visual Processing: Multi-scale edge and blob filters

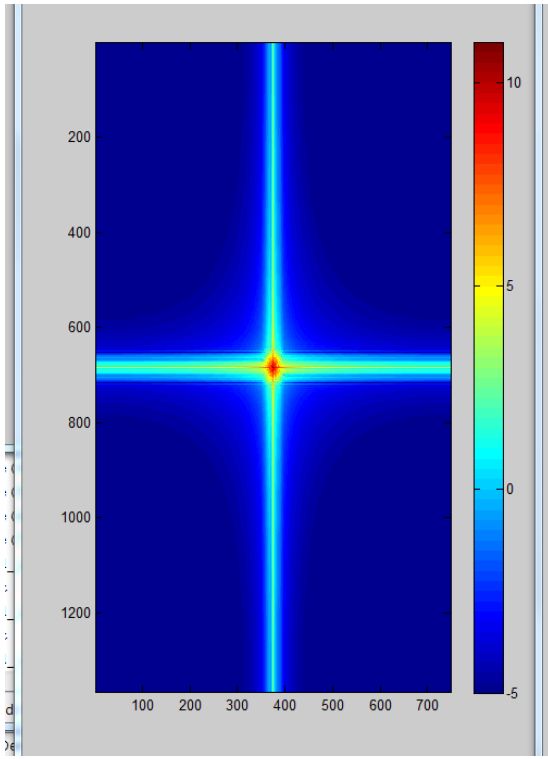
Hybrid Image in FFT

Hybrid Image



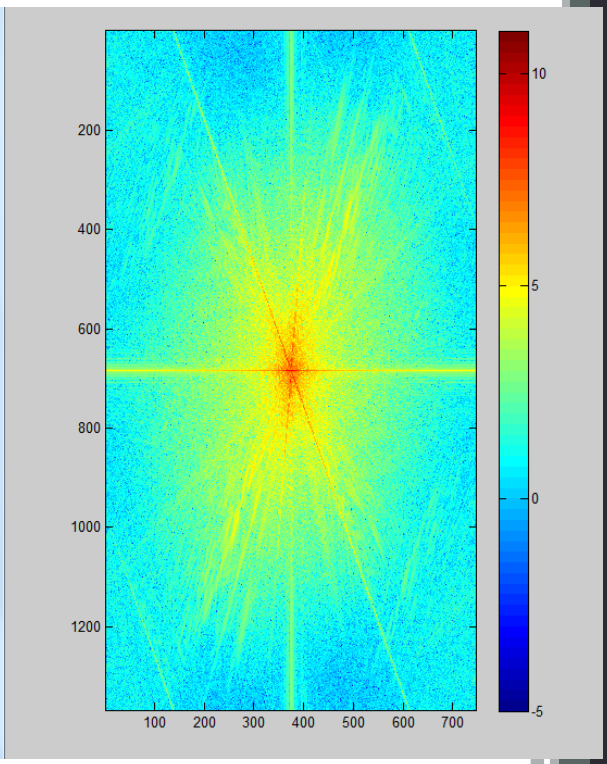
=

Low-passed Image

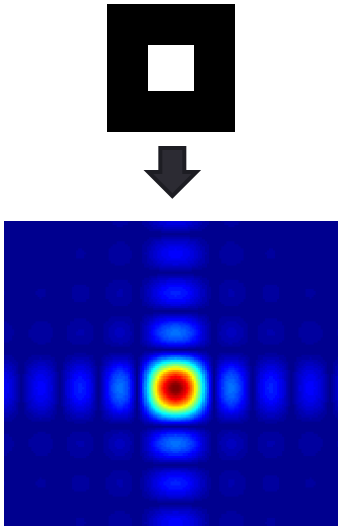


+

High-passed Image

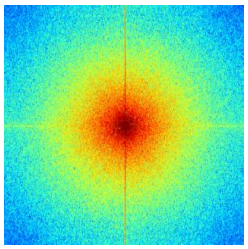


Things to Remember



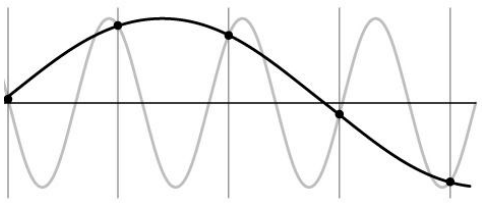
- Sometimes it makes sense to think of images and filtering in the frequency domain
  - Fourier analysis

- Can be faster to filter using FFT for large images (N logN vs. N^2 for auto-correlation)



- Images are mostly smooth
  - Basis for compression

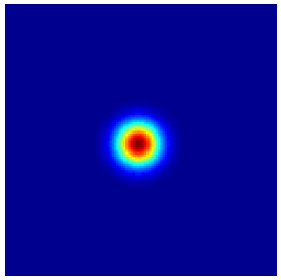
- Remember to low-pass before sampling



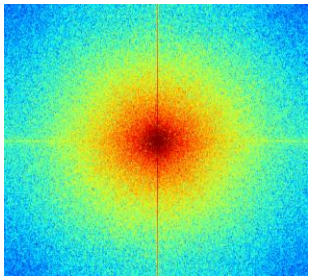
Practice question

1. Match the spatial domain image to the Fourier magnitude image

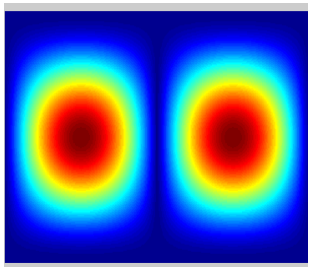
1



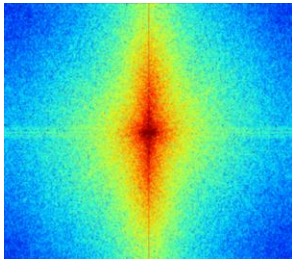
2



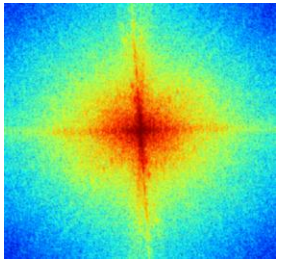
3



4



5

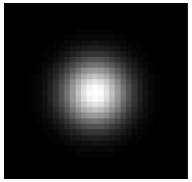


A

B

C

D



E

Next class

- Template matching
- Image Pyramids
- Filter banks and texture
- Denoising, Compression